SNR Maximization at CSI Aware AF Relay Assisted Networks

V. Golzadeh, S. Akhlaghi and M. Baghani
Electrical & Electronic Eng. Depart., Shahed University, Tehran, Iran
Emails: golzadeh, akhlaghi, baghani @shahed.ac.ir:
Corresponding author: S. Akhlaghi

***Abstract*— This paper concerns two-hop communication over relay-assisted Block fading channel. It is assumed there is not a direct link between the transmitter and the affiliated destination and the communication occurs in two hops through the use of a relay, where the amplify and forward (AF) strategy is employed at this node. In this case, in a Rayleigh block fading channel, the optimal weight function at the relay in terms of maximizing the received signal to noise ratio (SNR) at the destination is derived, assuming the relay is aware of channel gains associated with both hops and that the relay is subject to an average power constraint. Finally, the resulting SNR is compared to the case of having a peak power constraint at the relay which is served as a benchmark throughout the simulations.**

***Index Terms*—** amplify and forward relay, peak power constraint, average power constraint.

I. INTRODUCTION

It is widely recognized that relaying can greatly improve the quality of wireless communication links. In this regard, there have been some attempts to incorporate effective strategies at the relays, among them, the amplify and forward (AF) strategy is extensively addressed in the literature as there is no need to do sophisticated processing, while still having an acceptable performance as compared to other methods [1]–[3]. This strategy is first addressed in [8] and then in studied by many researchers.

This paper concerns communication in a two-hop block Rayleigh fading environment, where the channel gains are constant throughout one transmission block and vary independently for the next blocks. Moreover, we assume the relay knows the channel strengths associated with both hops.

This assumption has some practical implications and is extensively used in the literature [4], [5].

Most of existing works consider the relay is subject to a peak power constraint, and that it operates at full power. However, another realistic assumption is to have an average power constraint at this node. This is more useful in block fading channels, as the relay can save its power when the channel associated with the second hop is in poor condition, so that transmit at higher power when the channel associated with the second hop is in good condition. This gives rise to increasing the average signal to noise ratio (SNR) at the destination.

Manuscript Received 8-August-2012 and revised 5-October-2012 *ISSN: 2322-3936*
Accepted on 1- November-2012

To the best of authors' knowledge, this problem is not investigated elsewhere. However, there are some attempts to investigate the optimum power allocation strategies in terms of maximizing the sum-rate capacity or outage probability tradeoff which falls outside the scope of the current work [9].

In what follows, Section II presents the system model. Section III investigates the case when the relay has a peak power constraint. Section IV presents the proposed method, considering the relay is subject to an average power constraint. Section V aims to explore the results in Rayleigh block fading channels. Finally, Section VI illustrates the results and Section VII concludes the paper.

II. SYSTEM MODEL

Considering all nodes are equipped with single-antenna, the received signal at the relay can be written as,

$$y_r(t) = h_s x_s(t) + n_r(t) \quad \text{for } t = 1, \dots, N, \tag{1}$$

where $x_s(t)$ $(E[|x_s(t)|^2] = P_s)$ and $y_r(t)$ denote, respectively, the transmitted signal and the received signal at the relay. Also, $n_r(t)$ is an additive white Gaussian noise with unit variance, i.e., $n_r(t) \sim \mathcal{N}_c(0,1)$. h_s is the fading coefficient of the first hop in the current transmission block, which is assumed to be constant for each block and varies independently for the next blocks.

Similarly, the signal received at the destination ($y_d(t)$) is a faded noisy version of transmitted signal at the relay ($x_r(t)$) as follows,

$$y_d(t) = h_r x_r(t) + n_d(t), \tag{2}$$

where $n_d(t)$ is an additive white Gaussian noise of unit power, i.e., $n_d(t) \sim \mathcal{N}_c(0,1)$., and h_r is the channel gain associated with the second hop. Moreover, the AF relaying is assumed to be employed at the relay, thus $x_r(t)$ is a scaled version of the signal received at this node, i.e., $x_r(t) = \gamma\, y_r(t)$, where γ has yet to be determined.

When there is a peak power constraint, the relay is natural to send at full power. In this case, the resulting SNR is analytically derived in [6]. However, when the limitation is just over the average power, to the best of authors' knowledge, the optimum power allocation strategy is not fully treated in the literature. This motivated us to investigate this scenario through finding the best power allocation strategy at the relay, leading to the maximum achievable SNR at the destination. In what follows, we are going to first address the achievable SNR under peak power constraint, then we will address the proposed power allocation strategy under average power constraint.

III. THE PEAK POWER CONSTRAINT ASSUMPTION AT THE RELAY

In the general form of AF relaying, the relay simply forwards a scaled version of its received signal to the destination. Such a strategy is discussed in [6]. In this case, the source transmits its information to the relay and the relay re-transmits a scaled version of its received signal through applying a multiplication factor to operate at full power. The scaling factor used in this relaying strategy, with the peak power constraint P_r, becomes [6],

$$\gamma = \sqrt{\frac{P_r}{Ps * |hs|^2 + 1}} \quad . \tag{3}$$

Using (3) and referring to (1) and (2), the received signal at the destination can be formulated as follows,

$$Y_d = \gamma \, h_r h_s x_s + \gamma h_r n_r + n_d \ , \tag{4}$$

which can equivalently be written as,

$$Y'_d(t) = \frac{\gamma h_r h_s}{\sqrt{\gamma^2 |h_r|^2 + 1}} x_s(t) + n'_d(t) \ , \tag{5}$$

where $n'_d(t) \sim \mathcal{N}_c(0,1)$. In this case, referring to (5), the two-hop channel has an equivalent single-hop model, with transmit power P_s, and fading power s_e as follows,

$$s_e = \frac{\gamma^2 s_r s_s}{\gamma^2 s_r + 1} = \frac{P_r s_r s_s}{P_r s_r + P_s s_s + 1} \ , \tag{6}$$

where in (6), $s_s = |h_s|^2$ and $s_r = |h_r|^2$ are fading powers associated with the first and the second hops, respectively. The cumulative distribution function (cdf) of the equivalent fading power s_e can be written as [6],

$$F_{s_e}(x) = \Pr(s_e < x) = \int \int_R dx_s dx_r f_{s_s}(x_s) f_{s_r}(x_r) \ , \tag{7}$$

where the integration region can be defined as follows,

$$R = \{x_r, x_s \in [0, \infty) \,|\, \frac{P_r x_r x_s}{P_r x_r + P_s x_s + 1} \leq x\} \ . \tag{8}$$

Then, the cdf of s_e is given by

$$F_{s_e}(x) = 1 - \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \int_{\frac{x(1+P_r x_r)}{x_r P_r - x P_s}}^{\infty} dx_s f_{s_s}(x_s) f_{s_r}(x_r) \ . \tag{9}$$

Finally, taking derivation of $F_{s_e}(x)$ with respect to x, gives the corresponding probability density function(pdf) of the equivalent fading power, i.e.,

$$f_{s_e}(x) = \frac{d}{dx} F_{s_e}(x) \ . \tag{10}$$

IV. THE PROPOSED APPROACH WITH THE AVERAGE POWER CONSTRAINT AT THE RELAY

This section is motivated by the assumption that the relay has the ability to adjust its transmit power so that to improve the resulting SNR at the destination. In fact, it is assumed the relay's instantaneous transmit power is a function of the channel strengths associated with both hops. The problem is to find the best power allocation strategy such that the resulting SNR is maximized, assuming the relay is subject to an average power constraint. To this end, referring to (3), the amplification factor at the relay changes to,

$$\gamma = \sqrt{\frac{P_r(s_r,\ s_s)}{Ps * |hs|^2 + 1}}\ . \tag{11}$$

Similarly, the equivalent fading power s_e can be computed as,

$$s_e = \frac{\gamma^2 s_r s_s}{\gamma^2 s_r + 1} = \frac{s_r s_s P_r(s_r, s_s)}{s_r P_r(s_r,\ s_s) + s_s P_s + 1}\ . \tag{12}$$

Note that the objective is to find the optimal power allocation function $P_r(\ s_r, s_s)$ leading to the maximum fading power in (12) as maximizing s_e is equivalent to increasing the overall SNR. This problem can be cast as the following optimization problem,

$$\max \int_0^\infty \int_0^\infty \frac{x_r x_s P(x_r, x_s)}{x_r P(x_r, x_s) + x_s P_s + 1} f_{s_s}(x_s) f_{s_r}(x_r) dx_s dx_r$$
$$\text{s.t.} \int_0^\infty \int_0^\infty P(x_r, x_s) f_{s_s}(x_s) f_{s_r}(x_r) dx_s dx_r \le P_r\ . \tag{13}$$

Mathematically speaking, the problem is to find the function $P_r(x_r, x_s)$ from the set of functions which meet the constraint of (13), such that the equivalent fading power s_e is maximized. To this end, using the method of Lagrange multipliers, the optimization problem in (13) and its constraint can be encapsulated into the following single letter formula,

$$L(P_r) = \int_0^\infty \int_0^\infty \left(\frac{x_r x_s P(x_r, x_s)}{x_r P(x_r, x_s) + x_s P_s + 1} - \lambda P(x_r, x_s) \right) f_{s_s}(x_s) f_{s_r}(x_r) dx_s dx_r\ . \tag{14}$$

Defining $\mathcal{S}$ as the integrand of (14), i.e.,

$$\mathcal{S} = \frac{s_r s_s P(s_r, s_s)}{s_r P(s_r, s_s) + s_s P_s + 1} f_{s_s}(s_s) f_{s_r}(s_r) - \lambda P(s_r, s_s) f_{s_s}(s_s) f_{s_r}(s_r)\ , \tag{15}$$

the best value of $P = P_r(s_r, s_s)$ can be effectively computed through the use of the Euler equation [7] as follows,

$$\frac{\partial \mathcal{S}}{\partial P} = \frac{s_r s_s (s_s P_s + 1)}{(s_r P(s_r, s_s) + s_s P_s + 1)^2} f_{s_s}(s_s) f_{s_r}(s_r) - \lambda f_{s_s}(s_s) f_{s_r}(s_r) = 0\ . \tag{16}$$

Solving (16) yields,

$$P(s_r, s_s) = \frac{1}{s_r} \left(\sqrt{\frac{s_r s_s (s_s P_s + 1)}{\lambda}} - (s_s P_s + 1) \right). \tag{17}$$

Referring to (17) and noting the relay's instantaneous power is a positive function, it turns out that

s_r should meet the following constraint,

$$s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s} \quad , \tag{18}$$

otherwise, $P_r(s_r, s_s) = 0$. As a result, the average power constraint can be formulated as follows,

$$\int_0^{\infty} \int_{s_r \geq \frac{\lambda(s_s P_s+1)}{s_s}}^{\infty} \frac{1}{x_r} \left(\sqrt{\frac{x_r x_s (x_s P_s + 1)}{\lambda}} - (x_s P_s + 1) \right) f_{s_s}(x_s) f_{s_r}(x_r) dx_s dx_r = P_r \ . \tag{19}$$

Using the power allocation strategy defined in (17) at the relay and noting the constraint in (18), the cdf of the equivalent fading power s_e can be computed as,

$$F_{s_e}(x) = \Pr(s_e < x | s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s}) = \frac{\Pr(s_e < x \ \cap \ s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s})}{\mathrm{pr}\left(s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s}\right)} \quad , \tag{20}$$

Thus, the cdf of s_e can be rewritten as,

$$F_{s_e}(x) = \frac{\int\int_R f_{s_r}(x_r) f_{s_s}(x_s) dx_r dx_s}{\mathrm{pr}\left(s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s}\right)} \quad . \tag{21}$$

where noting the numerator of (20), the integration region R in (21) can be defined as follows,

$$R = \{x_r, x_s \in [0, \infty) | \frac{x_r x_s P(x_r, x_s)}{x_r P(x_r, x_s) + x_s P_s + 1} \leq x \ \cap x_r \geq \frac{\lambda(x_s P_s + 1)}{x_s}\} \ . \tag{22}$$

On the other hand, substituting $P_r(s_r, s_s)$ from (17) into (22) and after some mathematics, we arrive at the following,

$$(x_s - x)\sqrt{\frac{x_r x_s}{\lambda}} \leq x_s \sqrt{x_s P_s + 1}$$
$$x_r \geq \frac{\lambda(x_s P_s + 1)}{x_s} \quad . \tag{23}$$

It should be noted that the first constraint is relaxed when $x_s < x$; otherwise, the first constraint simplifies to $x_r \leq \frac{\lambda x_s (x_s P_s + 1)}{(x_s - x)^2}$. Noting this, the numerator of (21) can be computed as,

$$\int\int_R f_{s_r}(x_r) f_{s_s}(x_s) dx_r dx_s =$$
$$\int_0^x \int_{\frac{\lambda(x_s P_s+1)}{x_s}}^{\infty} f_{s_r}(x_r) f_{s_s}(x_s) dx_r dx_s + \int_x^{\infty} \int_{\frac{\lambda(x_s P_s+1)}{x_s}}^{\frac{\lambda x_s (x_s P_s+1)}{(x_s - x)^2}} f_{s_r}(x_r) f_{s_s}(x_s) dx_r dx_s \ . \tag{24}$$

V. RAYLEIGH FADING ENVIRONMENT

This section aims to explore the results in Rayleigh fading environment. It is assumed the channel strengths associated with both hops are i.i.d. with pdf $f_{s_r}(x_r) = e^{-x_r}$ and $f_{s_s}(x_s) = e^{-x_s}$. In what follows, we are going to address the resulting SNR of peak power constraint and average power constraint approaches.

A. The Peak power constraint approach

In such an environment, (9) simplifies to,

$$F_{s_e}(x) = 1 - \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \int_{\frac{x(1+P_r x_r)}{x_r P_r - x P_s}}^{\infty} dx_s e^{-x_s} e^{-x_r}$$

$$= 1 - \int_{\frac{P_s}{P_r}x}^{\infty} dx_r e^{-x_r - \frac{x(1+P_r x_r)}{x_r P_r - x P_s}} \quad . \tag{25}$$

As a result, taking derivation of (25) with respect to s_e, we arrive at the following,

$$f_{s_e}(x) = \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \frac{x_r P_r (1 + P_r x_r)}{(x_r P_r - x P_s)^2} e^{-x_r - \frac{x(1+P_r x_r)}{x_r P_r - x P_s}} \quad . \tag{26}$$

B. The proposed average power constraint approach

In this case, referring to (19), we note that λ can be numerically computed from the following constraint,

$$\int_0^{\infty} \int_{\frac{\lambda(x_s P_s + 1)}{x_s}}^{\infty} \frac{1}{x_r} \left(\sqrt{\frac{x_r x_s (x_s P_s + 1)}{\lambda}} - (x_s P_s + 1) \right) e^{-x_s} e^{-x_r} dx_r dx_s = P_r \quad . \tag{27}$$

It is worth mentioning that the left hand side of (27) is monotonically decreasing function with respect to λ as the integrand of the inner integral as well as the region of integration decreases as λ increases. Thus, there should be one λ which meets the constraint of (27).

Moreover, $Pr(s_e < x \cap s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s})$ can be computed as,

$$Pr\left(s_e < x \cap s_r \geq \frac{\lambda(s_s P_s + 1)}{s_s}\right) = \int \int_R e^{-x_s} e^{-x_r} dx_r dx_s$$

$$= \int_0^x \int_{\frac{\lambda(x_s P_s + 1)}{x_s}}^{\infty} e^{-x_s} e^{-x_r} dx_r dx_s + \int_x^{\infty} \int_{\frac{\lambda(x_s P_s + 1)}{x_s}}^{\frac{\lambda x_s (x_s P_s + 1)}{(x_s - x)^2}} e^{-x_s} e^{-x_r} dx_r dx_s$$

$$= \int_0^{\infty} e^{-\frac{\lambda(x_s P_s + 1)}{x_s}} e^{-x_s} dx_s + \int_x^{\infty} e^{-\frac{\lambda x_s (x_s P_s + 1)}{(x_s - x)^2}} e^{-x_s} dx_s \quad . \tag{28}$$

Substituting (28) in (21), it follows,

$$F_{s_e}(x) = 1 - \frac{\int_x^{\infty} e^{-\frac{\lambda x_s (x_s P_s + 1)}{(x_s - x)^2}} e^{-x_s} dx_s}{\int_0^{\infty} e^{-\frac{\lambda (x_s P_s + 1)}{x_s}} e^{-x_s} dx_s} \quad . \tag{29}$$

Using this, the corresponding pdf of s_e can be computed as,

$$f_{s_e}(x) = \frac{\int_x^{\infty} \frac{2\lambda x_s (x_s P_s + 1) e^{-\frac{\lambda x_s (x_s P_s + 1)}{(x_s - x)^2}} e^{-x_s}}{(x_s - x)^3} dx_s}{\int_0^{\infty} e^{-\frac{\lambda (x_s P_s + 1)}{x_s}} e^{-x_s} dx_s} \quad . \tag{30}$$

VI. NUMERICAL RESULTS

This section provides some numerical results, demonstrating the superiority of the proposed method under average power constraint over the conventional AF relaying with peak power constraint. Fig.I is provided to compare the cdf of the equivalent channel strength (s_e) associated with the proposed method with that of the conventional AF relaying under peak power constraint, showing the associated cdf of the proposed method is lower than the conventional form. This means, the probability that the equivalent channel strength falls below a certain threshold, is lower than the conventional AF relaying. Please note that to derive the associated CDF of equivalent channel between transmitter and the receiver, λ is numerically derived from (26) and then equation (29) is numerically computed. Also, substituting λ into (30) one can numericallycompute the corresponding pdf which can be used to obtain the average SNR at the destination.

Table I is also provided to show the average improvement in the SNR gain for different values of relay's power through the use of the proposed power allocation strategy as compared to the conventional AF approach.

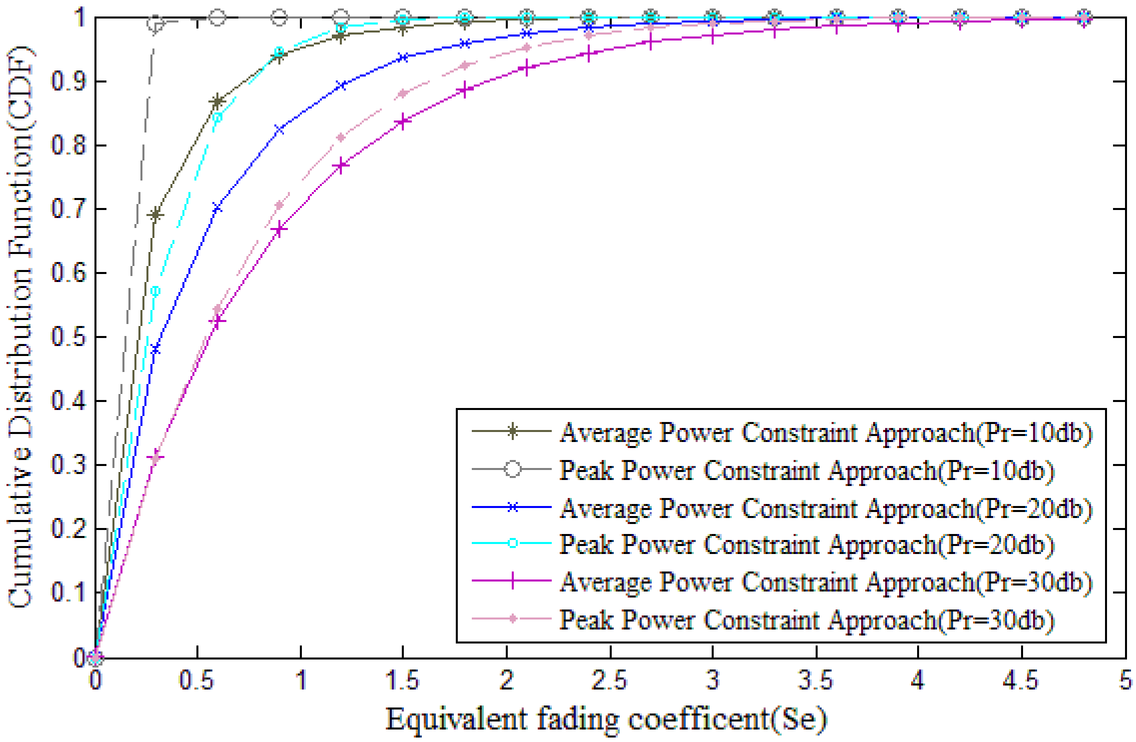

Fig. I. Comparison cdf curves associated with the Average Power constraint and Peak Power Constraint approaches for $\boldsymbol{Pr = 10dB, 20dB, 30dB}$ when the source power is set to $\boldsymbol{Ps = 20dB}$.

TABLE I. THE IMPROVED AVERAGE $\boldsymbol{SNR}$ BY THE USE OF THE PROPOSED METHOD AS COMPARED TO THE CONVENTIONAL AF RELAYING FOR DIFFERENT VALUES OF RELAY'S POWER, WHEN THE SOURCE POWER IS SET TO $\boldsymbol{P_s = 20dB}$.

Relay Power	**Improved SNR at destination(dB)**
10dB	5.9159
15dB	3.5052
20dB	1.8674
25dB	0.9436
30dB	0.4768

VII. CONCLUSION

This paper concerns two-hop communication with the aid of an AF relay in the middle. Accordingly, when the relay is subject to an average power constraint, the optimum power allocation strategy at the relay in terms of maximizing the received SNR is devised, showing the new method yields better result as compared to the conventional AF relaying which operates at full power regardless the channel condition. Numerical results for Rayleigh block fading channel demonstrates the superiority of the proposed method.

ACKNOWLEDGEMENT

The authors would like to greatly acknowledge the financial support provided by the Research Institute for ICT, Tehran, Iran.

REFERENCES

[1] B. Schein and R. G. Gallager, "The Gaussian parallel relay network," *IEEE Int. Symp. Inform. Theory (ISIT)*, p. 22, June 2000.

[2] M. Mohseni S. Zahedi and A. El Gamal, "On the capacity of AWGN relay channels with linear relaying functions," *IEEE Intern. Symp. Inform.Theory (ISIT)*, vol. 51, no. 3, pp. 399, 2004.

[3] M. Gastpar and M. Vetterli, "On the capacity of large Gaussian relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 3, pp. 765–779, 2005.

[4] Y. Zhang, Yi Ma and R.Tafazolli, "Modulation-adaptive cooperation schemes for wireless networks," in *Vehicular Technology Conference*, 2008, pp. 1320 –1324.

[5] H. B¨olcskei, R. U. Nabar, ¨O . Oyman and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. on wireless Communications*, vol. 5, no. 6, pp. 1433–1444, Jun. 2006.

[6] A. Steiner and S. Shamai (Shitz), "Single-user broadcasting protocols over a two-hop relay fading channel," *IEEE Trans. on Inform. Theory*, vol. 52, no. 11, pp. 4821 4838, Nov 2006.

[7] I. M. Gelfand and S. V. Formin, *Calculus of Variations*, Englewood Cliffs, NJ: Prentice-Hall, 1963

[8] J. N. Laneman and G. W. Wornell, "Exploiting Distributed Spatial Diversity in Wireless Networks ," in in Proc. Allerton Conf. Commun.,Contr., Computing, Illinois, Oct 2000.

[9] Y. Zhang, Y. Ma, R. Tafazolli, "Power Allocation for Bidirectional AF relaying over Rayleigh fading channels", IEEE Comm. Letters, vol. 14, no. 2, pp. 145-147, Feb 2010.