Outage Probability Bound and Diversity Gain for Ultra-Wideband Multiple-Access Relay Channels with Correlated Noises

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***Abstract*— In this paper, Ultra-wideband (UWB) multiple access relay channel with correlated noises at the relay and receiver is investigated. We obtain outer and inner bounds for the IEEE 802.15.3a UWB multiple access relay channel, and also, a diversity gain bound. Finally, we evaluate some results numerically and show that noise correlation coefficients play important role in determining relay position.**

***Index Terms*— Ultra wideband, multiple access relay channel, diversity gain, outage probability.**

I. INTRODUCTION

Information theoretic performance analysis of UWB communication systems is of practically importance, due to UWB extremely high data rates and diversity, coexistence capability with other wireless networks, accurate position location and ranging, no significant multipath fading, multiple access, covert communications and possible easier material penetration.

Possibly extension of discrete alphabet channels results to continuous alphabet versions has been of practically and theoretically importance. For example in addition to widely used Gaussian Shannon channel, there are many works such as Costa theorem [1] as the Gaussian version of discrete alphabet Gelfand-Pinsker theorem [2] and many other works related to fading or Gaussian version of discrete alphabet relay channels. In [3], the discrete alphabet degraded relay channel has been extended to Gaussian version. In [5] and [6], the previous results for discrete alphabets and memory less relay channel have been extended to UWB relay channel. In [5], authors derive bounds on the expected capacity and outage capacity of a three-node relay network with independent noises for UWB communications. In [6], a general achievable rate, two special capacity results and the max-flow min-cut outer bound for the UWB relay channel with correlated noises at the relay and destination are obtained.

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Our work: We obtain outer and inner bounds for the IEEE 802.15.3a UWB multiple access relay channel with correlated noises at the relay and receiver. We also obtain outage probability bound and diversity gain bound for UWB multiple access relay channel. At the last, we evaluate some results numerically.

Notation: Throughout the paper $Re(.)$, $\mathbb{E}(.)$, $var(.)$ and $cov(.)$ denote real part, expectation, variance and covariance operations, respectively. $\lfloor x \rfloor$ returns the largest integer $\leq x$. $diag(.)$ builds a diagonal matrix and $C(x) \triangleq \log(1+x)$ when *x* is complex.

II. PRELIMINARIES

In this section, we review relay channel, multiple access relay channel and IEEE UWB channel model. We introduce IEEE ultra-wideband multiple access relay channel at the end of this section.

A. Relay Channel (RC)

The RC is a three terminal channel consisting of a source node, a destination node and one node called the relay. The role of the relay node is to improve the overall performance of the communication between the source and destination such as the coverage area and ransmission rate.

The RC consists of four finite sets: $\mathcal{X}_R$, $\mathcal{X}_1$, $\mathcal{Y}_D$, and $\mathcal{Y}_R$ and a collection of conditional probability mass function $p(y_D, y_R \mid x_R, x_1)$ on $\mathcal{Y}_D \times \mathcal{Y}_R$, for all $(x_R, x_1) \in \mathcal{X}_R \times \mathcal{X}_1$; x_1 and x_R are the channel inputs, which are sent by the transmitter and the relay, respectively; and y_D and y_R are the channel outputs of the receiver and the relay. The channel is assumed to be memoryless and also the relay encoder is supposed to be strictly causal, which means that the RC input x_R at a given moment (t) depends only on the past relay observations of the transmitted messages, which is written as,

$$x_{R,t} = f_t(y_R^{t-1}), \qquad t = 1, 2, \cdots, n \tag{1}$$

A $(2^{nR}, n)$ code for the RC consists of a set of integers $\mathcal{W}_1 = \{1, 2, \cdots, 2^{nR}\}$, an encoding function that maps each message $w_1 \in \mathcal{W}_1$ into a codeword, $x_1 : \mathcal{W}_1 \rightarrow \mathcal{X}_1^n$ and a set of relay functions $\{f_t\}_{t=1}^n$ such that $x_{R,t} = f_t(y_R^{t-1})$, $1 \leq t \leq n$ and a decoding function $g : \mathcal{Y}_D^n \rightarrow \mathcal{W}_1$. A rate R is achievable if there exists a sequence of $(2^{nR}, n)$ codes with $P_e^{(n)} = \frac{1}{2^{nR}} \Sigma_{w_1 \in \mathcal{W}_1} Pr\{g(Y_D^n) \neq w_1 \mid w_1 \text{ sent}\} \xrightarrow{n\rightarrow\infty} 0$, assuming a uniform distribution over the messages. Channel capacity C is defined as the supremum over the set of achievable rates.

Furthermore, another important characteristic of RC is the employed relaying strategy, which can be partial decode and forward (PDF), decode and forward (DF), compress and forward (CF), amplify

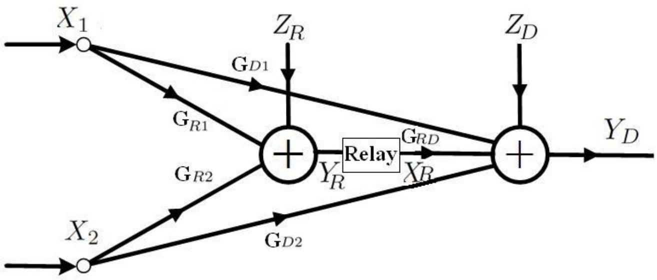

Fig. 1. UWB Multiple-access relay channel

and forward (AF) and noisy network coding.

The RC with PDF and DF Strategies: In PDF strategy, the relay decodes some information (U, auxiliary random variable) of the message sent by the source, where U may be a part or an index of the message. In DF strategy, the relay decodes the whole of message ($U = X_1$) and cooperates with the sender to help the destination in decoding. This strategy is close to optimal when the source-relay channel is excellent, which practically happens when the source and relay are physically near each other. In this work, relay uses the DF strategy like [11].

B. Multiple-Access Relay Channel

1) Discrete Memory less Multiple-Access Relay Channel:

The 2-source discrete memoryless multiple-access relay channel consists of 3 inputs; $X_k\,(k=1,2)$, and X_R from the sources and the relay, respectively, and two output Y_R, and Y_D at the relay and receiver, respectively. This model is defined by $\{(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R), p(y_R, y_D \mid x_1, x_2, x_R), \mathcal{Y}_R \times \mathcal{Y}_D\}$, where $\mathcal{X}_1, \mathcal{X}_2$ and $\mathcal{X}_R$ are the input alphabets; $\mathcal{Y}_R$ and $\mathcal{Y}_D$ are the output alphabets. This model might fit a situation in wireless sensor networks where sensors (the sources) are too weak to cooperate, but they can send their data to more powerful nodes that form a “backbone” network [7].

C. IEEE UWB Channel Model

IEEE 802.15.4a group published a channel model for UWB communications [8]. The channel is modeled as a linear system with an impulse response as follows,

$$h(t) = \tilde{\beta} \sum_{l=0}^{L-1} \sum_{i=0}^{M-1} a_{i,l} e^{j\phi_{i,l}} \delta(t - T_l - \tau_{i,l}) \tag{2}$$

where T_l and $\tau_{i,l}$ represents the cluster and ray arrival times, respectively, and they have Poission distributions. The factor $\tilde{\beta}$ jointly models the pathloss, shadowing and antenna insertion loss. $a_{i,l}$ is the gain of the i^{th} path in the l^{th} cluster and finally $\phi_{i,l}$ is the complex baseband phase of each multipath component. L is the number of clusters and M is the number of rays in each cluster.

The complex baseband communication system can be represented as a discrete-time vector channel where the transmitter sends a complex vector $x=(x_0,\cdots,x_{K-1})^T$ and the destination receives an output vector $y=(y_0,\cdots,y_{K-1})^T$ given by,

$$y_i=\sum_{k=0}^{K'-1} g_k x_{i-k}+z_i, \qquad i=0,\cdots,K-1 \tag{3}$$

where,

$$g_k=\sum_{i,l:\lfloor d_{i,l}/T_s\rfloor=k} \tilde{\beta}\alpha_{i,l}e^{-j2\pi f_c d_{i,l}}, \qquad (d_{i,l}=T_l+\tau_{i,l}, T_s=\frac{1}{W}) \tag{4}$$

and $z=(z_0,\cdots,z_{K-1})^T$ is complex Gaussian with circularly symmetric independent components $z_i\sim\mathcal{CN}(0,N)$ and K' is the memory length. Since $\{\alpha_{i,l}\}$ are zero-mean and uncorrelated; therefore, $\{g_k\}$ are zero-mean and uncorrelated. The channel state vector $\mathbf{g}$ stays fixed within each block of data transmission and change in an independent and identically distributed fashion from one block to another, also we assume that the communication is coherent, i.e., the receiver knows the $\{g_k\}$. The size of each block, K, is constrained by the channel coherence time and can be at most equal to $\frac{T_c}{T_s}$ where T_c is coherent time and T_s is sampling period that it is inverse of the bandpass channel bandwidth. In [9], a frequency domain model of the above UWB channel is obtained by taking DFT from both sides of (3) as follows,

$$Y_i=G_iX_i+Z_i, \qquad i=0,1,\cdots,K-1 \tag{5}$$

where the vectors $\mathbf{G}$ are the DFT of vector of complex baseband channel coefficients $\mathbf{g}=(g_0,g_1,\cdots,g_{K-1})$.

D. UWB Multiple Access Relay Channel Model

$\mathbf{X}_1=(\mathbf{X}_{1,0},\cdots,\mathbf{X}_{1,K-1})^T$, $\mathbf{X}_2=(\mathbf{X}_{2,0},\cdots,\mathbf{X}_{2,K-1})^T$ denote the K-point DFT of the transmitted UWB signals from sender 1 and sender 2 to relay and destination and and $\mathbf{X}_R=(\mathbf{X}_{R,0},\cdots,\mathbf{X}_{R,K-1})^T$ denotes the K-point DFT of the transmitted UWB signals from relay to destination. And similarly $\mathbf{Y}_R=(\mathbf{Y}_{R,0},\cdots,\mathbf{Y}_{R,K-1})^T$ and $\mathbf{Y}_D=(\mathbf{Y}_{D,0},\cdots,\mathbf{Y}_{D,K-1})^T$ represent the DFT of the received signals at relay and destination, respectively. Then by this frequency domain model, we can formulate the input-output relation of UWB-MARC as,

$$Y_{Ri} = (G_{R1i}X_{1i} + G_{R2i}X_{2i}) + Z_{Ri}, \qquad i = 0,1,\cdots,K-1 \tag{6}$$

$$Y_{Di} = (G_{D1i}X_{1i} + G_{D2i}X_{2i} + G_{RDi}X_{Ri}) + Z_{Di}, \qquad i = 0,1,\cdots,K-1 \tag{7}$$

where the noise terms $\{Z_{Ri}\}$ and $\{Z_{Di}\}$ are *i.i.d* with $\sim \mathcal{CN}(0, N_R)$ and $\sim \mathcal{CN}(0, N_D)$, respectively for the i^{th} received sample. The vectors $\mathbf{G}$ are the DFT of vectors of complex baseband channel coefficients $\mathbf{g} = (g_0, \cdots, g_{K-1})^T$ related to each link. This model is shown in Fig. 1.

III. MAIN THEOREMS

In this section, we obtain and prove two main theorems. In one of them, we obtain an outer bound and in another, we obtain an inner bound for UWB multiple access relay channel.

A. UWB multiple access relay channel inner bound:

An achievable rate region for K-block delay constrained multiple-access relay channel is given by [11]:

$$\bigcup \{(R_1, R_2):$$

$$\sum_{t\in S} R_t \le \min\Big(\frac{1}{K}\sum_{i=0}^{K-1} I(X_i, X_{Ri}; Y_{Di} \mid V_{S_i^C}, X_{S_i^C}), \frac{1}{K}\sum_{i=0}^{K-1} I(X_{Si}; Y_{Ri} \mid V_{1i}, V_{2i}, X_{S_i^C}, X_{Ri})\Big)\Big\} \tag{8}$$

where $S \subseteq \{1,2\}$, S^C is complement of S in set $\{1,2\}$ and the union is taken over all

$$p(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_R, \mathbf{v}_1, \mathbf{v}_2, \mathbf{y}_D, \mathbf{y}_R) = \prod_{i=0}^{K-1} p(y_{Di}, y_{Ri} \mid x_{1i}, x_{2i}, x_{Ri})\, p(v_{1i})\, p(v_{2i})\, p(x_{Ri} \mid v_{1i}, v_{2i}) \prod_{k=1}^{2} p(x_{ki} \mid v_{ki}) \tag{9}$$

where V_{1i} and V_{2i} are independent random variables with finite alphabets to help the sources cooperate with the relay. Now, we extend these results to the UWB version in the following theorems.

Theorem1. A delay-constrained general achievable rate region for frequency selective block fading ultra-wideband multiple-access relay channel is given by:

$$\bigcup \{(R_1, R_2):$$

$$\sum_{t\in S} R_t \le \max_{\alpha_{1i},\alpha_{2i},\beta_{1i},\beta_{2i},i=0,\ldots,K-1} \min\Big(\frac{1}{K}\sum_{i=0}^{K-1} C(\frac{\theta_{1i}}{N_R}), \frac{1}{K}\sum_{i=0}^{K-1} C(\frac{\theta_{2i}}{N_D})\Big) \tag{10}$$

where,

$$\theta_{1i} = \sum_{t\in S} |G_{Rti}|^2 \beta_{ti} P_t$$

$$\theta_{2i} = \sum_{t\in S} (|G_{RDi}|^2 \alpha_{ti} P_R + |G_{Dti}|^2 P_t + 2\sqrt{P_t P_R}\,\mathrm{Re}\{G_{Dti} G_{RDi}^* \sqrt{\alpha_{ti}\bar{\beta}_{ti}}\})$$

and $S \subseteq \{1,2\}$ and S^C is complement of S in set $\{1,2\}$.

Proof :

We assume that the sources and relay nodes transmit their signals per complex baseband sample with following constraints:

$$\frac{1}{K}\sum_{i=0}^{K-1}|X_{1i}|^2 \le P_1, \quad \frac{1}{K}\sum_{i=0}^{K-1}|X_{2i}|^2 \le P_2, \quad \frac{1}{K}\sum_{i=0}^{K-1}|X_{Ri}|^2 \le P_R$$

Let $X_{Ri} \sim \mathcal{CN}(0,P_R), X_{1i} \sim \mathcal{CN}(0,P_1), X_{2i} \sim \mathcal{CN}(0,P_2), V_{1i} \sim \mathcal{CN}(0,1), V_{2i} \sim \mathcal{CN}(0,1)$, $M_{1i} \sim \mathcal{CN}(0,\beta_{1i}P_1), M_{2i} \sim \mathcal{CN}(0,\beta_{2i}P_2)$ where M_{1i}, M_{2i}, V_{1i} and V_{2i} are mutually independent. We generate random variables X_{1i}, X_{2i} and X_{Ri} according to (9) and as following way,

$$X_{Ri} = \sqrt{P_R}(\sqrt{\alpha_{1i}}V_{1i} + \sqrt{\alpha_{2i}}V_{2i}) \tag{11}$$

$$X_{ki} = M_{ki} + \sqrt{\bar{\beta}_{ki}P_k}V_{ki}, \qquad k = 1,2 \tag{12}$$

where M_{1i} and M_{2i} carry the fresh information and V_1, V_2 and the relay input coherently carry the refinement information. $|\beta_{ki}| = 1 - |\bar{\beta}_{ki}|$ and $|\alpha_{1i}| + |\alpha_{2i}| = 1$. These coefficient distribute the sources power between fresh information and refinement information.

Now, we generate random code as following:

1) Generate 2^{nR_k} independent identically distributed $\mathbf{V_k}$, according to $\mathcal{CN}(0,\mathbf{I})$ and index them as $\mathbf{V_k}(m_k)$, $m_k \in [1:2^{nR_k}], k = 1,2$.
2) Generate 2^{nR_k} independent identically distributed $\mathbf{M_k}$, according to $\mathcal{CN}(0,\mathbf{C}_{Mk})$ and index them as $\mathbf{M_k}(w_k)$, $w_k \in [1:2^{nR_k}]$ and $\mathbf{C}_{Mk} = diag(\beta_{k0}P_k, \cdots, \beta_{k(K-1)}P_k), k = 1,2$.
3) For each $\mathbf{V_k}(m_k)$, generate 2^{nR_k} conditionally independent $\mathbf{X_k}$. Index them as $\mathbf{X_k}(w_k \mid m_k)$, $w_k \in [1:2^{nR_k}]$ where $\mathbf{X_k}(w_k \mid m_k) = \sqrt{P_k}[\sqrt{\bar{\beta}_0}, \cdots, \sqrt{\bar{\beta}_{K-1}}] \times \mathbf{V_k}(m_k) + \mathbf{M_k}(w_k)$.
4) For each $\{\mathbf{V_1}(m_1), \mathbf{V_2}(m_2)\}$, choose one $\mathbf{X_R}$. Index them as $\mathbf{X_R}(m_1,m_2)$, $m_k \in [1:2^{nR_k}]$, where $\mathbf{X_R}(m_1,m_2) = \sqrt{P_R}[\sqrt{\alpha}_{10}, \cdots, \sqrt{\alpha}_{1,K-1}] \times \mathbf{V_1}(m_1) + \sqrt{P_R}[\sqrt{\alpha}_{2,0}, \cdots, \sqrt{\alpha}_{2,K-1}] \times \mathbf{V_2}(m_2)$

where $\times$ denotes an element by element matrix multiplication.

For brevity, we prove only two expressions.

$$\begin{aligned} I(X_{1i};Y_{Ri} \mid V_{1i},V_{2i},X_{Ri},X_{2i}) &= h(Y_{Ri} \mid V_{1i},V_{2i},X_{Ri},X_{2i}) - h(Y_{Ri} \mid V_{1i},V_{2i},X_{Ri},X_{1i},X_{2i}) \\ &= h((G_{R1i}X_{1i} + G_{R2i}X_{2i}) + Z_{Ri} \mid V_{1i},V_{2i},X_{Ri},X_{2i}) \\ &\quad - h(Y_{Ri} \mid V_{1i},V_{2i},X_{Ri},X_{1i},X_{2i}) \end{aligned}$$

$$= h(G_{R1i}X_{1i} + Z_{Ri} \,|\, V_{1i}, V_{2i}, X_{Ri}, X_{2i}) - h(Z_{Ri})$$

$$= h(G_{R1i}(M_{1i} + \sqrt{\bar{\beta}_{1i}P_1}V_{1i}) + Z_{Ri} \,|\, V_{1i}, V_{2i}, X_{Ri}, X_{2i}) - h(Z_{Ri})$$

$$= h(G_{R1i}M_{1i} + Z_{Ri} \,|\, V_{1i}, V_{2i}, X_{Ri}, X_{2i}) - h(Z_{Ri})$$

$$= \log 2\pi e(|G_{R1i}|^2 \beta_{1i}P_1 + N_R) - \log 2\pi e(N_R)$$

Also, we have:

$$I(X_{1i}, X_{2i}, X_{Ri}; Y_{Di}) = h(Y_{Di}) - h(Y_{Di} \,|\, X_{1i}, X_{2i}, X_{Ri})$$

$$= \log 2\pi e(|G_{D1i}|^2 P_1 + |G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R$$

$$+2\sqrt{P_1P_R}\,\mathrm{Re}\{G_{D1i}G^*_{RDi}\sqrt{\alpha_{1i}\bar{\beta}_{1i}}\} + 2\sqrt{P_2P_R}\,\mathrm{Re}\{G_{D2i}G^*_{RDi}\sqrt{\alpha_{2i}\bar{\beta}_{2i}}\} + N_D) - \log 2\pi e(N_D)$$

$$I(X_{1i}, X_{2i}, X_{Ri}; Y_{Di})$$

$$= C\Big(\frac{|G_{D1i}|^2 P_1 + |G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R + 2\sqrt{P_1P_R}\,\mathrm{Re}\{G_{D1i}G^*_{RDi}\sqrt{\alpha_{1i}\bar{\beta}_{1i}}\} + 2\sqrt{P_2P_R}\,\mathrm{Re}\{G_{D2i}G^*_{RDi}\sqrt{\alpha_{2i}\bar{\beta}_{2i}}\}}{N_D}\Big)$$

B. *UWB multiple access relay channel outer bound:*

Theorem 2. A K- block delay constrained form for the max-flow min-cut outer bound on the capacity region of the multiple access relay channel can be expressed as [11]:

$$\bigcup\{(R_1, R_2):$$

$$\sum_{t\in S} R_t \le \max_{\alpha_{1i},\alpha_{2i},\beta_{1i},\beta_{2i}, i=0,\dots,K-1} \min\Big(\frac{1}{K}\sum_{i=0}^{K-1} C(\frac{\phi_{1i}}{N_D}), \frac{1}{K}\sum_{i=0}^{K-1} C(\phi_{2i})\Big) \qquad (13)$$

where,

$$\phi_{1i} = \sum_{t\in S} |G_{Dti}|^2 P_1 t + |G_{RDi}|^2 P_R + 2\sqrt{P_tP_R}\,\mathrm{Re}\{G_{Dti}G^*_{RDi}\sqrt{\bar{\beta}_{ti}\bar{\alpha}_i} + \varsigma_i\}$$

$$\phi_{2i} = \sum_{t\in S} P_t \frac{(1-\bar{\beta}_{ti}\bar{\alpha}_i)}{1-|\rho_{zi}|^2}\Big(\frac{|G_{Dti}|^2}{N_D} + \frac{|G_{Rti}|^2}{N_R} - 2\frac{\mathrm{Re}\{G_{Rti}G^*_{Dti}\,|\rho_{zi}|\}}{\sqrt{N_DN_R}}\Big)$$

and $S \subseteq \{1,2\}$ and S^C is complement of S in set $\{1,2\}$. Also, $\varsigma_i = 0$ when $S = \{1\}$ or $S = \{2\}$ and $\varsigma_i = 2\sqrt{P_{1i}P_{2i}}\{G_{D1i}G_{D2i}\sqrt{\bar{\beta}_{1i}\bar{\beta}_{2i}}\}$ when $S = \{1,2\}$. $\rho_{zi} = \frac{\mathbb{E}(Z_{Di}Z_{Ri})}{\sqrt{N_DN_R}}$ is correlation coefficient between Z_{Di} and Z_{Ri}.

Corollary 1. As seen easily, the outer bound in [12] is obtained form (13) by removing one sender.

Proof : The proof is a direct consequence of the max-flow min-cut theorem. The cut sets for MARC are illustrated in Fig. 2. Across the cut C_1, the maximum rate of information transfer is bounded by:

$$R_1 \le \frac{1}{K}\sum_{i=0}^{K-1} I\left(X_{1i};Y_{Di},Y_{Ri} \mid X_{2i},X_{Ri}\right)$$

where,

$$I\left(X_{1i};Y_{Di},Y_{Ri} \mid X_{2i},X_{Ri}\right) = h(Y_{Di},Y_{Ri} \mid X_{2i},X_{Ri}) - h(Y_{Di},Y_{Ri} \mid X_{1i},X_{2i},X_{Ri})$$

$$\le h(Y_{Di},Y_{Ri} \mid X_{Ri}) - h(Y_{Di},Y_{Ri} \mid X_{1i},X_{2i},X_{Ri})$$

$$\le \log((2\pi e)^2(\det cov(Y_{Di},Y_{Ri} \mid X_{Ri}))) - \log((2\pi e)^2(\det cov(Z_{Di},Z_{Ri})))$$

$$= \log((2\pi e)^2 \det\begin{pmatrix} \mathbb{E}(Y_{Di}^2 \mid X_{Ri}) & \mathbb{E}(Y_{Di}Y_{Ri}^* \mid X_{Ri}) \\ \mathbb{E}(Y_{Ri}Y_{Di}^* \mid X_{Ri}) & \mathbb{E}(Y_{Ri}^2 \mid X_{Ri}) \end{pmatrix}$$

$$-\log((2\pi e)^2(\det cov(Z_{Di},Z_{Ri})))$$

$$= \log((2\pi e)^2 \det\begin{pmatrix} |G_{D1i}|^2 P_1(1-\rho_{1Ri}^2)+N_D & G_{R1i}G_{D1i}^*P_1(1-\rho_{1Ri}^2)+\sqrt{N_RN_D}\rho_{zi} \\ G_{R1i}^*G_{D1i}P_1(1-\rho_{1Ri}^2)+\sqrt{N_RN_D}\rho_{zi}^* & |G_{R1i}|^2 P_1(1-\rho_{1Ri}^2)+N_R \end{pmatrix}$$

$$-\log((2\pi e)^2 N_R N_D (1-\rho_{zi}^2)$$

where,

$\rho_{1Ri} = \dfrac{\mathbb{E}(X_{1i}X_{Ri})}{\sqrt{P_1P_R}} = \sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}$. So, we have,

$$I\left(X_{1i};Y_{Di},Y_{Ri} \mid X_{2i},X_{Ri}\right) \le \log\left(1+P_1\frac{(1-\bar{\beta}_{1i}\bar{\alpha}_i)}{1-|\rho_{zi}|^2}\left(\frac{|G_{D1i}|^2}{N_D}+\frac{|G_{R1i}|^2}{N_R}-2\frac{\mathrm{Re}\{G_{R1i}G_{D1i}^* \mid \rho_{zi}|\}}{\sqrt{N_DN_R}}\right)\right)$$

Similarly, by considering C_2,

$$I\left(X_{2i};Y_{Di},Y_{Ri} \mid X_{1i},X_{Ri}\right) \le \log\left(1+P_2\frac{(1-\bar{\beta}_{2i}\bar{\alpha}_i)}{1-|\rho_{zi}|^2}\left(\frac{|G_{D2i}|^2}{N_D}+\frac{|G_{R2i}|^2}{N_R}-2\frac{\mathrm{Re}\{G_{R2i}G_{D2i}^* \mid \rho_{zi}|\}}{\sqrt{N_DN_R}}\right.\right.$$

Now, by considering C_1', we have

$$R_1 \le \frac{1}{K}\sum_{i=0}^{K-1} I\left(X_{1i},X_{Ri};Y_{Di} \mid X_{2i}\right)$$

where,

$$I\left(X_{1i},X_{Ri};Y_{Di} \mid X_{2i}\right) = h(Y_{Di} \mid X_{2i}) - h(Y_{Di} \mid X_{1i},X_{2i},X_{Ri})$$

$$= \log(2\pi e(|G_{D1i}|^2 P_1 + |G_{RDi}|^2 P_R + 2\sqrt{P_1P_R}\,\mathrm{Re}\{G_{D1i}G_{RDi}^*\sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}\} + N_D))$$

$$-\log(2\pi e N_D)$$

So, we have

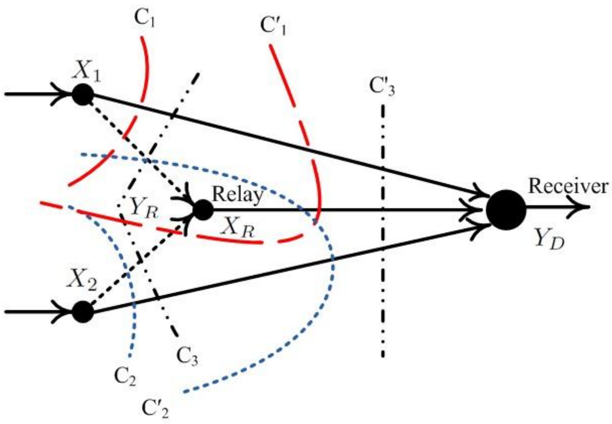

Fig. 2. Illustration of cut sets for MARC

$$I\left(X_{1i}, X_{Ri}; Y_{Di} \mid X_{2i}\right) = \log(1 + \frac{|G_{D1i}|^2 P_1 + |G_{RDi}|^2 P_R + 2\sqrt{P_1 P_R}\,\mathrm{Re}\{G_{D1i} G_{RDi}^* \sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}\}}{N_D})$$

Similarly, by considering C_2', we have:

$$I\left(X_{2i}, X_{Ri}; Y_{Di} \mid X_{1i}\right) = \log(1 + \frac{|G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R + 2\sqrt{P_2 P_R}\,\mathrm{Re}\{G_{D2i} G_{RDi}^* \sqrt{\bar{\beta}_{2i}\bar{\alpha}_i}\}}{N_D})$$

Now, by considering C_3, we have

$$R_1 + R_2 \le \frac{1}{K}\sum_{i=0}^{K-1} I\left(X_{1i}, X_{2i}; Y_{Di}, Y_{Ri} \mid X_{Ri}\right)$$

where,

$$I\left(X_{1i}, X_{2i}; Y_{Di}, Y_{Ri} \mid X_{Ri}\right) = h(Y_{Di}, Y_{Ri} \mid X_{Ri}) - h(Y_{Di}, Y_{Ri} \mid X_{1i}, X_{2i}, X_{Ri})$$

$$\le \log((2\pi e)^2 (\det cov(Y_{Di}, Y_{Ri} \mid X_{Ri}))) - \log((2\pi e)^2 (\det cov(Z_{Di}, Z_{Ri})))$$

So, we have

$$I\left(X_{1i}, X_{2i}; Y_{Di}, Y_{Ri} \mid X_{Ri}\right) \le \log(1 + A_i''' + B_i''')$$

where,

$$A_i''' = P_1 \frac{(1-\bar{\beta}_{1i}\bar{\alpha}_i)}{1-|\rho_{zi}|^2}\left(\frac{|G_{D1i}|^2}{N_D} + \frac{|G_{R1i}|^2}{N_R} - 2\frac{\mathrm{Re}\{G_{R1i} G_{D1i}^* |\rho_{zi}|\}}{\sqrt{N_D N_R}}\right)$$

$$B_i''' = P_2 \frac{(1-\bar{\beta}_{2i}\bar{\alpha}_i)}{1-|\rho_{zi}|^2}\left(\frac{|G_{D2i}|^2}{N_D} + \frac{|G_{R2i}|^2}{N_R} - 2\frac{\mathrm{Re}\{G_{R2i} G_{D2i}^* |\rho_{zi}|\}}{\sqrt{N_D N_R}}\right)$$

Lastly, across the cut C_3', the maximum sum rate of information transfer is bounded by:

$$R_1 + R_2 \le \frac{1}{K}\sum_{i=0}^{K-1} I\left(X_{1i}, X_{2i}, X_{Ri}; Y_{Di}\right)$$

$$\begin{aligned} I\left(X_{1i}, X_{2i}, X_{Ri}; Y_{Di}\right) &= h(Y_{Di}) - h(Y_{Di} \mid X_{1i}, X_{2i}, X_{Ri}) \\ &= \log(2\pi e(|G_{D1i}|^2 P_1 + |G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R \\ &\quad + 2\sqrt{P_1 P_2}\operatorname{Re}\{G_{D1i}G_{D2i}^* \bar{\alpha}_i \sqrt{\bar{\beta}_{1i}\bar{\beta}_{2i}}\} + 2\sqrt{P_1 P_R}\operatorname{Re}\{G_{D1i}G_{RDi}^* \sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}\} \\ &\quad + 2\sqrt{P_2 P_R}\operatorname{Re}\{G_{D2i}G_{RDi}^* \sqrt{\bar{\beta}_{2i}\bar{\alpha}_i}\} + N_D) - \log(2\pi e N_D) \end{aligned}$$

and,

$$\begin{aligned} I\left(X_{1i}, X_{2i}, X_{Ri}; Y_{Di}\right) &= h(Y_{Di}) - h(Y_{Di} \mid X_{1i}, X_{2i}, X_{Ri}) \\ &= \log(2\pi e(|G_{D1i}|^2 P_1 + |G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R \\ &\quad + 2\sqrt{P_1 P_2}\operatorname{Re}\{G_{D1i}G_{D2i}^* \bar{\alpha}_i \sqrt{\bar{\beta}_{1i}\bar{\beta}_{2i}}\} + 2\sqrt{P_1 P_R}\operatorname{Re}\{G_{D1i}G_{RDi}^* \sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}\} \\ &\quad + 2\sqrt{P_2 P_R}\operatorname{Re}\{G_{D2i}G_{RDi}^* \sqrt{\bar{\beta}_{2i}\bar{\alpha}_i}\} + N_D) - \log(2\pi e N_D) \end{aligned}$$

So, we have

$$I\left(X_{D1i}, X_{D2i}, X_{Ri}; Y_{Di}\right) \le \log\left(1 + \frac{A_i'' + B_i''}{N_D}\right)$$

where,

$$A_i'' = |G_{D1i}|^2 P_1 + |G_{D2i}|^2 P_2 + |G_{RDi}|^2 P_R$$

$$\begin{aligned} B_i'' &= \sqrt{P_1 P_2}\operatorname{Re}\{G_{D1i}G_{D2i}^* \bar{\alpha}_i \sqrt{\bar{\beta}_{1i}\bar{\beta}_{2i}}\} \\ &\quad + 2\sqrt{P_1 P_R}\operatorname{Re}\{G_{D1i}G_{RDi}^* \sqrt{\bar{\beta}_{1i}\bar{\alpha}_i}\} + 2\sqrt{P_2 P_R}\operatorname{Re}\{G_{D2i}G_{RDi}^* \sqrt{\bar{\beta}_{2i}\bar{\alpha}_i}\} \end{aligned}$$

IV. OUTAGE PROBABILITY ANALYSIS

Outage probability: Outage probability is an important measure of performance of communication systems such as mobile systems and can be used as a minimum quality of service requirement. The outage probability implies that there is a nonzero probability that a given transmission rate cannot be supported by the channel.

A. Statistical properties of the channel frequency coefficients

If a_k is a random variable with Nakagami distribution, then gamma probability density of a_k^2 is given by [13]:

$$p(a_k^2) = (\frac{m_k}{\Omega_k})^2 \frac{a^{2m_k-2}}{\Gamma(m_k)} exp(-\frac{m_k}{\Omega_k} a^2) \quad a^2 \geq 0, m_k \geq 0.5 \tag{14}$$

where the mean and variance of a_k^2 are given by

$$\mathbb{E}[a_k^2] = \Omega_k, \qquad Var[a_k^2] = \frac{\Omega_k^2}{m_k} \tag{15}$$

The sum of L independent gamma terms, $s = \sum_{k=1}^{L} a_k^2$, was approximated by an equivalent gamma distribution with the following parameters [14]:

$$\Omega_s \simeq \sum_{k=1}^{L} \Omega_k, \qquad m_s \simeq \frac{(\sum_{k=1}^{L} \Omega_k)^2}{\sum_{k=1}^{L} \frac{\Omega_k^2}{m_k}} \tag{16}$$

Lemma 1. The linear combination of independent gamma terms, $s_\alpha = \sum_{k=1}^{L} \alpha_k a_k^2$, can be approximated by an equivalent gamma distribution with the following parameters:

$$\Omega_{(s_\alpha)} \simeq \sum_{k=1}^{L} \alpha_k \Omega_k, \qquad m_{(s_\alpha)} \simeq \frac{(\sum_{k=1}^{L} \alpha_k \Omega_k)^2}{\sum_{k=1}^{L} \alpha_k^2 \frac{\Omega_k^2}{m_k}} \tag{17}$$

Proof : The proof is omitted for brevity.

We assume that the multipath components arrive at uniform delays, so g_k in (4) reduces to one term

$$g_k = \tilde{\beta} a_{i,l} e^{j\phi_{i,l}} \Big|_{i,l: \lfloor (T_l + \tau_{i,l})/T_s \rfloor = k} \tag{18}$$

Therefore, $\mathbb{E}\{g_k\}$ becomes zero. As $\mathbf{G}$ are the DFT of $\mathbf{g}$; consequently, $\mathbb{E}\{G_i\} = 0$. Also, $\mathbf{G}_{D1}, \mathbf{G}_{D2}, \mathbf{G}_{R1}, \mathbf{G}_{R2}$ and $\mathbf{G}_{RD}$ are independent, so $\mathbb{E}\{G_{mi} G_{ni}\} = 0$ for each $m, n = D1, D2, R1, R2, RD$.

The channel coefficient $a_{i,l}$ has Nakagami distribution ([8]) with parameters $m_{i,l}$ and

$$\Omega_{i,l} = \frac{1}{\gamma_l [(1-\beta)\gamma_1 + \beta\gamma_2 + 1]} e^{-\frac{\tau_{i,l}}{\gamma_l}} e^{-\frac{T_l}{\Gamma_l}} 10^{\frac{M_{cluster}}{10}} \tag{19}$$

in which T_l is the arrival time of the l^{th} cluster and $\tau_{i,l}$ is the arrival time of the i^{th} ray in the l^{th} cluster relative to the cluster arrival time T_l. $M_{cluster}$ (cluster shadowing) is a zero mean normally distributed variable with standard deviation $\sigma_{cluster}$ (cluster shadowing variance), γ_l and Γ_l are the intra-cluster decay time constant and mean energy of each cluster, respectively, and γ_1, γ_2 are the ray arrival rates, and β is mixing factor ([8]).

And from [5], we have:

$$\mathbb{E}\{\sum_{l=0}^{L-1}\sum_{i=0}^{M-1}|a_{i,l}|^2\} = \xi exp(\frac{(ln(10))^2}{200}\sigma_{cluster}^2)\kappa^{LOS} \cdot \frac{\phi'^M - 1}{\phi' - 1}\frac{\mu' e^{\bar{L}(\mu'-1)} - 1}{\mu' - 1} \tag{20}$$

where $\phi' = \frac{\beta\gamma_1}{\gamma_1 + \frac{1}{\gamma_0}} + \frac{(1-\beta)\gamma_2}{\gamma_2 + \frac{1}{\gamma_0}}$, $\mu' = \frac{\gamma}{\gamma + \frac{1}{\Gamma}}$ (γ is inter-cluster arrival rate ([8]), $\bar{L}$ denotes the mean number of clusters, $\xi = \frac{1}{\gamma_0[(1-\beta)\gamma_1 + \beta\gamma_2 + 1]}$ and κ^{LOS} is equal to 1 for LOS connection and equal to μ' for NLOS connection. Using the γ_0 is for omitting the dependency of γ_l and ϕ_l' to l ([5],[8]).

The variance of g_k can be calculated as follows [15]:

$$\begin{aligned} var\left(\sum g_{k_{real}}\right) &= var\left(\sum_l\sum_i \tilde{\beta} a_{i,l} e^{j\phi_{i,l}}\right)_{real} \\ &= \frac{1}{2}\sum_l\sum_i\left(\mathbb{E}\left\{|\tilde{\beta}\, a_{i,l}|^2\right\}\right) \\ &= \frac{1}{2}\sum_l\sum_i\left(\mathbb{E}\left\{|\tilde{\beta}|^2\right\}\mathbb{E}\left\{|a_{i,l}|^2\right\}\right) \\ &= \frac{\xi}{2PL} exp(\frac{(ln(10))^2}{200}\sigma_{cluster}^2)\kappa^{LOS} \\ &\quad \times \frac{\phi'^M - 1}{\phi' - 1}\frac{\mu' e^{\bar{L}(\mu'-1)} - 1}{\mu' - 1} \end{aligned} \tag{21}$$

where PL is the pathloss and it is assumed that β models only the pathloss effect and the effect of shadowing and antenna insertion loss are neglected. According to the Parseval's relation, we have

$$\sum_{k=0}^{K-1}|G_{m,k}|^2 = \sum_{k=0}^{K-1}|g_{m,k}|^2 = \sum_{k=0}^{(K_m'-1)}|g_{m,k}|^2 \simeq \frac{1}{2PL_m}\sum_{i,l}|a_{i,l}^{(m)}|^2 \tag{22}$$

PL_m is the pathloss of the m^{th} link. K_m' is the ISI length for m^{th} link ($m = D1, D2, R1, R2, RD$). For example, in path between relay and receiver, we can write the following expression,

$$\sum_{k} |g_{RD,k}|^2 \simeq \frac{1}{2PL_{RD}} \sum_{i,l} |a_{i,l}^{(RD)}|^2 = \frac{\xi_{RD}}{2PL_{RD}} exp(\frac{(ln(10))^2}{200} \sigma_{cluster,RD}^2) \kappa_{RD}^{LOS} \times \frac{\phi'^{M_{RD}} - 1}{\phi'_{RD} - 1} \frac{\mu'_{RD} e^{\bar{L}_{RD}(\mu'_{RD}-1)} - 1}{\mu'_{RD} - 1} \quad (23)$$

B. Outage Probability

In this section, we approximate the outer bound on the capacity region of the UWB multiple access relay channel (13) with using the law of large numbers and Jensen's inequality. Then, we obtain the outage probability. We can write an outer bound for the outage probability of the multiple access relay channel as follows,

$$P_{outage}^{UB} = P\{\bigcup (R_1, R_2) < \bigcup (R_{1,o}, R_{2,o})\}$$

$$\geq P\begin{pmatrix} \min\left(R_{11}^{max}, R_{12}^{max}\right) < R_{1,o} \\ \min\left(R_{21}^{max}, R_{22}^{max}\right) < R_{2,o} \\ \min\left(R_{sum,1}^{max}, R_{sum,2}^{max}\right) < R_{sum,o} \end{pmatrix}$$

$$\geq \begin{pmatrix} \max(P\{R_{11}^{max} < R_{1,o}\}, P\{R_{12}^{max} < R_{1,o}\}) \\ \max(P\{R_{21}^{max} < R_{2,o}\}, P\{R_{22}^{max} < R_{2,o}\}) \\ \max(P\{R_{sum,1}^{max} < R_{sum,o}\}, P\{R_{sum,2}^{max} < R_{sum,o}\}) \end{pmatrix}$$

Therefore,

$$P_{outage}^{UB} \geq \begin{pmatrix} \max(P_{11,outage}^{max}, P_{12,outage}^{max}) \\ \max(P_{21,outage}^{max}, P_{22,outage}^{max}) \\ \max(P_{sum1,outage}^{max}, P_{sum2,outage}^{max}) \end{pmatrix} \quad (24)$$

where $R_{1,o}$ and $R_{2,o}$ are individual target rates, $R_{sum,o}$ is sum target , $P_{ij,outage}^{max}$ denotes probability of $R_{ij}^{max} < R_{i,o}$ ([5],[13],[14]).

1) Outage probability of R_{11}^{max} :

We can write an outer bound on R_{11}^{max} as follows,

$$R_{11}^{max} = \frac{1}{K} \sum_{i=0}^{K-1} \log(1 + \frac{A+B}{N_D})$$

$$\overset{a}{\leq} \log(1 + \frac{\sum_{i=0}^{K-1} P_1 |G_{D1i}|^2 + \sum_{i=0}^{K-1} P_R |G_{RDi}|^2 + \sum_{i=0}^{K-1} 2\sqrt{P_1 P_R} \operatorname{Re}\{G_{D1i} G_{RDi}^*\}}{KN_D})$$

$$\overset{b}{\leq} \log(1 + \frac{\sum_{i=0}^{K-1} P_1 |G_{D1i}|^2 + \sum_{i=0}^{K-1} P_R |G_{RDi}|^2}{KN_D})$$

$$\overset{c}{=}\log(1+SNR_1(\sum_{i,l}|a_{i,l}^{(D1)}|^2+\gamma_{RD1}\sum_{i,l}|a_{i,l}^{(RD)}|^2) \quad (25)$$

where,

$$A=|G_{D1i}|^2 P_1+|G_{RDi}|^2 P_R$$

$$B=2\sqrt{P_1P_R}\,\mathrm{Re}\{G_{D1i}G_{RDi}^*\}$$

and

- \`a' follows from Jensen's inequality.
- \`b' follows from the law of large numbers. Since K is a very large number, we can approximate the term $\mathrm{Re}(\frac{1}{K}\sum_{i=0}^{K-1}\{G_{D1i}G_{RDi}^*\})$ with $\mathrm{Re}(\mathbb{E}\{G_{D1i}G_{RDi}^*\})$.
- \`c' follows from equality of transmitted powers, $SNR_1 \triangleq \frac{1}{2PL_{D1}}\frac{P_1}{KN_D}$, $\gamma_{RD1} \triangleq \frac{PL_{D1}}{PL_{RD}}\frac{P_R}{P_1}$ and (22).

So, we have:

$$\begin{aligned} P_{11,outage}^{max} &= P\{R_{11}^{max} < R_{1,o}\} \\ &= P\{\log(1+SNR_1(\sum_{i,l}|a_{i,l}^{(D1)}|^2+\gamma_{RD1}\sum_{i,l}|a_{i,l}^{(RD)}|^2)<R_{1,o}\} \\ &= P\{(\sum_{i,l}|a_{i,l}^{(D1)}|^2+\gamma_{RD1}\sum_{i,l}|a_{i,l}^{(RD)}|^2)<\frac{2^{R_{1,o}}-1}{SNR_1}\} \\ &= P\{(\Xi_{D1}+\gamma_{RD1}\Xi_{RD})<\Xi_{th1}\} \end{aligned} \quad (26)$$

where Ξ_{D1} and Ξ_{RD} are random variables with gamma distribution and parameters (m_{D1},Ω_{D1}) and (m_{RD},Ω_{RD}) which are defined with following parameters (15,16):

$$\Omega_{D1} \simeq \sum_{i,l}\Omega_{i,l}^{(D1)}, \qquad m_{D1} \simeq \frac{\Omega_{D1}^2}{\sum_{i,l}\frac{(\Omega_{i,l}^{(D1)})^2}{m_{i,l}}} \quad (27)$$

$$\Omega_{RD} \simeq \sum_{i,l}\Omega_{i,l}^{(RD)}, \qquad m_{RD} \simeq \frac{\Omega_{RD}^2}{\sum_{i,l}\frac{(\Omega_{i,l}^{(RD)})^2}{m_{i,l}}} \quad (28)$$

So, we can write:

$$P_{11,outage}^{max} = \frac{\gamma(m_{11}, \frac{m_{11}\Xi_{th1}}{\Omega_{11}})}{\Gamma(m_{11})} \tag{29}$$

where $\Gamma(.)$ is the gamma function and $\gamma(.,.)$ is the lower incomplete gamma function.

Using the *Lemma 1* and choosing $\alpha_1 = 1$ and $\alpha_2 = \gamma_{RD1}$, we can write the following relations:

$$m_{11} \simeq \frac{(\Omega_{D1} + \gamma_{RD1}\Omega_{RD})^2}{\frac{\Omega_{D1}^2}{m_{D1}} + \gamma_{RD1}^2 \frac{\Omega_{RD}^2}{m_{RD}}} \tag{30}$$

$$\Omega_{11} \simeq \Omega_{D1} + \gamma_{RD1}\Omega_{RD} \tag{31}$$

2) Outage probability of R_{12}^{max} :

Like section (IV-B1), we can write an outer bound on R_{12}^{max} as follows,

$$P_{12,outage}^{max} = P\{(\Xi_{D1} + \gamma_{D1R1}\Xi_{R1}) < \Xi_{th1}\} \tag{32}$$

where $\gamma_{D1R1} \triangleq \frac{PL_{D1}}{PL_{R1}} \frac{N_D}{N_R}$. Also, Ξ_{D1} and Ξ_{D1R1} are random variables with gamma distribution and parameters (m_{D1}, Ω_{D1}) and (m_{R1}, Ω_{R1}) which (m_{R1}, Ω_{R1}) is defined with following parameters (15,16):

$$\Omega_{R1} \simeq \sum_{i,l} \Omega_{i,l}^{(R1)}, \qquad m_{R1} \simeq \frac{\Omega_{R1}^2}{\sum_{i,l} \frac{(\Omega_{i,l}^{(R1)})^2}{m_{i,l}}} \tag{33}$$

So, we can write

$$P_{12,outage}^{max} = \frac{\gamma(m_{12}, \frac{m_{12}\Xi_{th1}}{\Omega_{12}})}{\Gamma(m_{12})} \tag{34}$$

Therefore,

$$\begin{aligned} P_{1,outage} &\geq \max(P_{11,outage}^{max}, P_{12,outage}^{max}) \\ &= \max\Big(\frac{\gamma(m_{11}, \frac{m_{11}\gamma_{th1}}{\Omega_{11}})}{\Gamma(m_{11})}, \frac{\gamma(m_{12}, \frac{m_{12}\gamma_{th1}}{\Omega_{12}})}{\Gamma(m_{12})}\Big) \end{aligned} \tag{35}$$

As the same way,

$$P_{2,outage} \geq \max(P_{21,outage}^{max}, P_{22,outage}^{max})$$
$$= \max\left(\frac{\gamma(m_{21}, \frac{m_{21}\Xi_{th2}}{\Omega_{21}})}{\Gamma(m_{21})}, \frac{\gamma(m_{22}, \frac{m_{22}\Xi_{th2}}{\Omega_{22}})}{\Gamma(m_{22})}\right) \quad (36)$$

3) Outage probability of $R_{sum,1}^{max}$:

We can write an outer bound on $R_{sum,1}^{max}$ as follows,

$$P_{sum1,outage}^{max} = \frac{\gamma(m_{sum,1}, \frac{m_{sum,1}\Xi_{th1}}{\Omega_{sum,1}})}{\Gamma(m_{sum,1})} \quad (37)$$

where,

$$m_{sum,1} \simeq \frac{\left(\Omega_{D1} + \frac{SNR_2}{SNR_1}\Omega_{D2} + \gamma_{RD1}\Omega_{RD}\right)^2}{\frac{\Omega_{D1}^2}{m_{D1}} + (\frac{SNR_2}{SNR_1})^2 \frac{\Omega_{D2}^2}{m_{D2}} + \gamma_{RD1}^2 \frac{\Omega_{RD}^2}{m_{RD}}} \quad (38)$$

$$\Omega_{sum,1} = \Omega_{D1} + \frac{SNR_2}{SNR_1}\Omega_{D2} + \gamma_{RD1}\Omega_{RD} \quad (39)$$

where $SNR2 \triangleq \frac{1}{2PL_{D2}} \frac{P_2}{KN_D}$. For the lack of space, proof is omitted.

Similarly, we can write an outer bound on $R_{sum,2}^{max}$ as follows,

$$P_{sum2,outage}^{max} = \frac{\gamma(m_{sum,2}, \frac{m_{sum,2}\Xi_{th,sum}}{\Omega_{sum,2}})}{\Gamma(m_{sum,2})} \quad (40)$$

where,

$$m_{sum,2} \simeq \frac{\left(\Omega_{D1} + \frac{SNR_2}{SNR_1}\Omega_{D2} + \gamma_{D1R1}\Omega_{R1} + \gamma_{D1R2}\Omega_{R2}\right)^2}{\frac{\Omega_{D1}^2}{m_{D1}} + (\frac{SNR_2}{SNR_1})^2 \frac{\Omega_{D2}^2}{m_{D2}} + \gamma_{D1R1}^2 \frac{\Omega_{R1}^2}{m_{R1}} + \gamma_{D1R2}^2 \frac{\Omega_{R2}^2}{m_{R2}}} \quad (41)$$

$$\Omega_{sum,2} = \Omega_{D1} + \frac{SNR2}{SNR1}\Omega_{D2} + \gamma_{D1R1}\Omega_{R1} + \gamma_{D1R2}\Omega_{R2} \quad (42)$$

Consequently,

$$P_{sum,outage} \geq \max(P_{sum1,outage}^{max}, \mathbb{P}_{sum2,outage}^{max})$$

$$= \max\left(\frac{\gamma(m_{sum,1},\frac{m_{sum,1}\Xi_{th,sum}}{\Omega_{sum,1}})}{\Gamma(m_{sum,1})},\frac{\gamma(m_{sum,1},\frac{m_{sum,2}\Xi_{th,sum}}{\Omega_{sum,2}})}{\Gamma(m_{sum,2})}\right) \quad (43)$$

Outage probability bound:

The following is an approximated bound for outage probability of the UWB multiple access relay channel:

$$P_{outage}^{UB} \geq \begin{pmatrix} \max\left(\frac{\gamma(m_{11},\frac{m_{11}\gamma_{th1}}{\Omega_{11}})}{\Gamma(m_{11})},\frac{\gamma(m_{12},\frac{m_{12}\gamma_{th1}}{\Omega_{12}})}{\Gamma(m_{12})}\right) \\ \max\left(\frac{\gamma(m_{21},\frac{m_{21}\Xi_{th2}}{\Omega_{21}})}{\Gamma(m_{21})},\frac{\gamma(m_{22},\frac{m_{22}\Xi_{th2}}{\Omega_{22}})}{\Gamma(m_{22})}\right) \\ \max\left(\frac{\gamma(m_{sum,1},\frac{m_{sum,1}\Xi_{th,sum}}{\Omega_{sum,1}})}{\Gamma(m_{sum,1})},\frac{\gamma(m_{sum,1},\frac{m_{sum,2}\Xi_{th,sum}}{\Omega_{sum,2}})}{\Gamma(m_{sum,2})}\right) \end{pmatrix} \quad (44)$$

C. Diversity Gain Analysis and Comparison

The $\mathbb{P}_{11,outage}^{max}$ gives us a diversity gain of m_{11} and $\mathbb{P}_{12,outage}^{max}$ gives us a diversity gain of m_{12}. and so on. Therefore, we can write the following bound for diversity gain,

$$Diversity \quad Gain \leq \begin{pmatrix} \min(m_{11}, m_{12}) \\ \min(m_{21}, m_{22}) \\ \min(m_{sum,1}, m_{sum,2}) \end{pmatrix} \quad (45)$$

For comparison, we assume that the number of clusters and the number of rays in each cluster and in all paths are equal. Also, all of paths are Nakagami variables with similar parameters Ω and m and with equal path loss. Noise power spectral density at the destination and the relay are assumed equal. It is assumed that the sources and relay power are equal ($P_1 = P_2 = P_R$). We can compute m_{11}, m_{12}, $\cdots$ as following,

$$\Omega_{D1} = K\Omega = LM\Omega, \qquad m_{D1} = Km = LMm$$

then

$$\Omega_{11} = K\Omega + \gamma_{RD1}K\Omega = LM\Omega + \gamma_{RD1}LM\Omega$$

by considering the above assumptions,

$$\Omega_{11} = 2K\Omega = 2LM\Omega$$

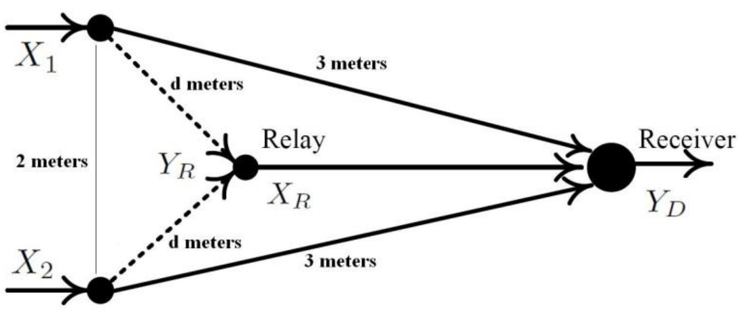

Fig. 3. Simulation model.

and,

$$m_{11} = 2Km = 2LMm$$

as the same way

$$m_{12} = m_{21} = m_{22} = 2Km = 2LMm$$

and,

$$m_{sum,1} = 3Km = 3LMm, \qquad m_{sum,2} = 4Km = 4LMm$$

Consequently,

$$(Diversity \quad Gain)_{UWB_{MARC}} \le LMm \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \tag{46}$$

By removing relay from (13), we obtain the diversity gain bound for UWB multiple access channels as follows,

$$(Diversity \quad Gain)_{UWB_{MAC}} \le LMm \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{47}$$

By comparing (46) with (47), we see an improvement in diversity gain when we add a relay to multiple access channel.

V. NUMERICAL RESULTS

In this section, we illustrate some numerical results for the derived bounds and regions. We examine our results in NLOS environment with 8 GHz bandwidth and center frequency of 6 GHz. The distance between senders and receiver is fix and about 3 meters but the distance between two senders

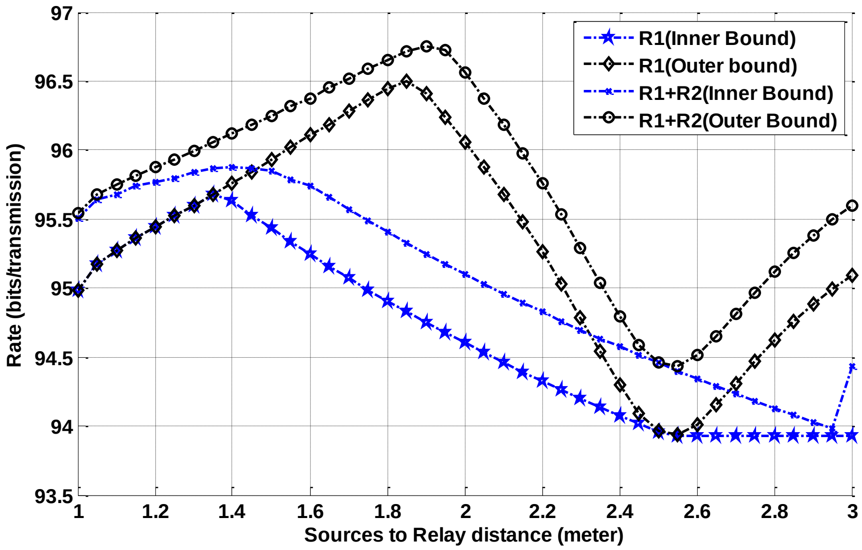

Fig. 4. Inner and outer bounds for R_1 and R_1+R_2 for $\rho_z=0.99$.

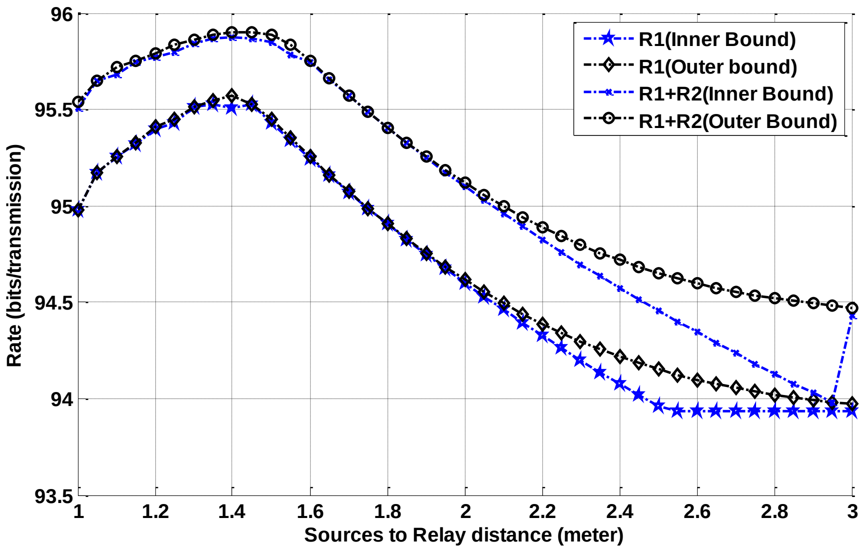

Fig. 5. Inner and outer bounds for R_1 and R_1+R_2 for $\rho_z=0.50$.

is 2 meters and the relay is located in distance of d meters from the both senders (Fig. 3). The transmitted power by the sources and relay nodes are equal to the maximum allowed power for the UWM systems, defined by FCC (-41.3dBm/MHz). We assume noise power spectral density at the destination is half of noise power spectral density at the relay ($N_D=0.5N_R$). In Figs. 4, 5 and 6 the inner and outer bounds for R_1 and R_1+R_2 for three different values of correlation coefficients (0.99, 0.50 and 0.00) are shown. We can say that our inner bound is independent of noise correlation coefficients, because of using decode and forward scheme. Based on the Fig. 7, we see that when the

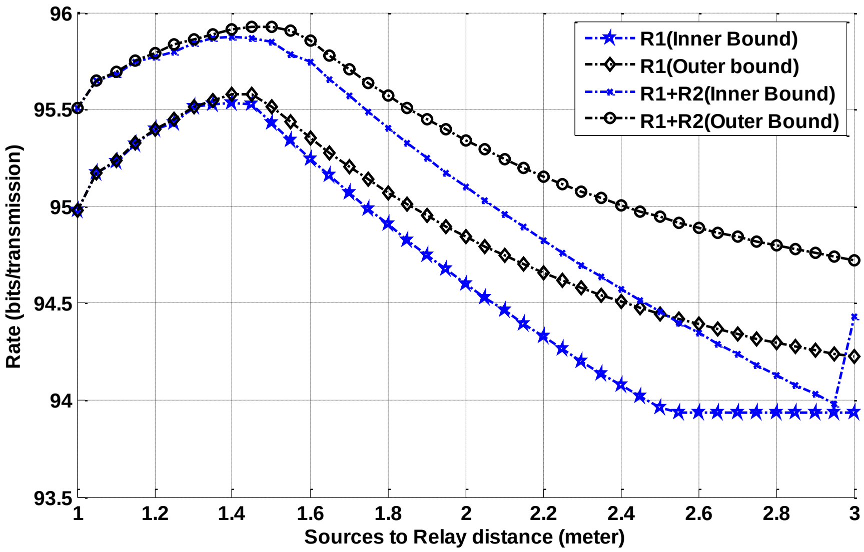

Fig. 6. Inner and outer bounds for R_1 and $R_1 + R_2$ for $\rho_z = 0$.

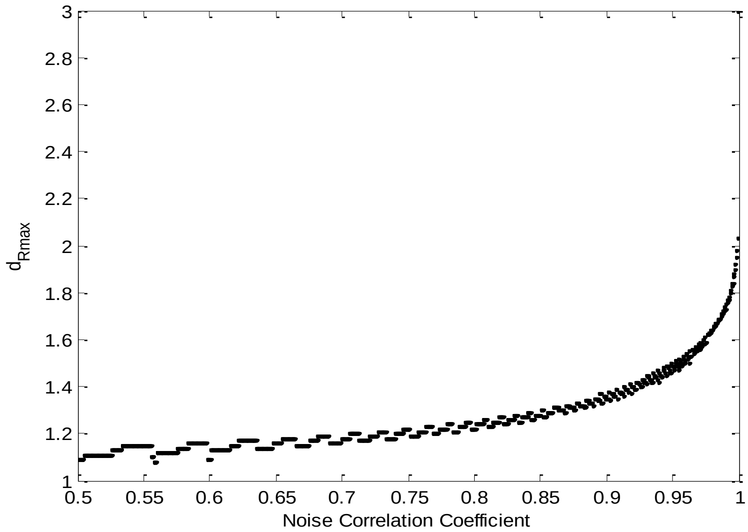

Fig. 7. Distance between relay and sources for maximum individual rates of outer bound.

noise correlation coefficient increases, the distance between relay and sources for maximum individual rates of outer bound is increased. Therefore, it is better that the relay is located near the sources when the noise correlation coefficient is low. Therefore, noise correlation coefficients plays important role in determining relay position and data rate. And also, as intuitively expected, the noise correlation coefficient appears in terms having (Y_D, Y_R). In achievable rate terms there is not (Y_D, Y_R) and hence we do not see the noise correlation coefficient. Physically speaking, we observe that achievable rate is not a function of the correlation coefficient, because the relay node performs full-decoding of the sources messages and send newly encoded messages.

VI. CONCLUSION

We have obtained outer and inner bounds for the IEEE 802.15.3a UWB multiple access relay channel with correlated noises at the relay and receiver. We have also obtained the diversity gain bound and shown that there is an improvement in diversity gain compared with UWB multiple access channels. Our model subsumes UWB relay channel with and without correlated noises. At the last, we evaluated some results numerically.

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