# A Low Complexity Forward Error Correction for PAPR reduction in OFDM Systems

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　　　　*Abstract*—**In this paper, a Peak to Average Power Ratio (PAPR) reduction technique in Orthogonal Frequency Division Multiplexing (OFDM) systems is proposed. This technique uses error correction capability of channel coding to reduce the PAPR. The coded bits at the input of OFDM modulator are separated into two groups of the Most Significant Bits (MSB) and the Least Significant Bits (LSB). The bits are mapped to the constellation points such that the MSBs are less affected by the channel error than the LSBs. In the proposed method, the MSBs are changed by an intentional error such that the PAPR is minimized. The performance of this method has been presented and it has been shown that it achieves the better performance than that of the Selected Mapping (SLM) method.**

## I. INTRODUCTION

The OFDM is an interesting multicarrier transmission technique for broadband wireless communications over multipath fading channels [1]. However one of the disadvantages of the OFDM systems is high PAPR of the transmitted signal. This causes clipping of the OFDM signal by the High Power Amplifier (HPA). Due to the nonlinear distortion, the efficiency of HPA is reduced and the complexity of Analog to Digital (A/D) and Digital to Analog (D/A) converters is increased [2]. Various techniques have been proposed in the literature to overcome this problem [1]-[12] .These methods can be classified as the distorting and distortion-less techniques [4]. The examples of the distorting techniques are clipping and filtering [5] and peak windowing [6]. The examples of the distortion-less techniques are constellation shaping [7], tone reservation [8], block coding technique [1],[9],[12] Selected Mapping (SLM) [10] and Partial Transmit Sequences (PTS) [11]. Among these methods, SLM and PTS need Side Information (SI) to be transmitted to the receiver side.

Channel coding can be used for PAPR reduction in OFDM systems. Two approaches are used in coding based PAPR reduction techniques. In the first approach, the code is designed such that at the output of the channel coding, only the codewords with low PAPR are generated. Davis and Jedwab showed that the PAPR is 3dB if the codewords are chosen from the Golay complementary sequences

[12]. But the main limitation is that the length of the Golay sequences is limited and it can not be used for OFDM systems with a large number of subcarriers [12]. In the second approach, the error correction capability of the channel coding is used for PAPR reduction. At the transmitter side some of the bits are affected by intentional error [1]. The locations of the inserted errors are selected such that the PAPR is minimized. It is obvious that this method degrades the Bit Error Rate (BER) of the receiver.

In this paper, a coding based technique for PAPR reduction is presented which is based on the second approach. The main advantage of the proposed technique is that in the determination of the error locations, the coding and mapping blocks are considered simultaneously. In fact the bits, that are less affected by the channel error, are used for PAPR reduction. The coded bits at the input of OFDM modulator are separated into two groups of the MSBs and LSBs. When the bits are mapped to the constellation points the MSBs determine the quadrant and the LSBs choose a constellation point in the selected quadrant. Since the MSBs are less affected by the channel error than the LSBs, the intentional errors are located in MSB positions.

In the next section, the system model of OFDM and the concept of PAPR are introduced. Section III describes the proposed technique for PAPR reduction. The simulation result and analysis of the performance of the proposed method are presented in section IV. In this section, the effect of the proposed method in PAPR reduction at the transmitter side and its effect on the BER at the receiver side are evaluated.

## II. PAPR OF OFDM SIGNAL

The time domain baseband representation of an OFDM signal with $N$ subcarriers can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i \, e^{j2\pi f_i t} \qquad t \in [0, T] \tag{1}$$

Where $j = \sqrt{-1}$ , $f_i = i\Delta f$, and $\Delta f = \frac{1}{T}$. $T$ is the OFDM symbol period and $\{X_i\}_{i=0}^{N-1}$ are data symbols (chosen from the constellation points). The time domain samples of OFDM signal with sampling rate of $\frac{LN}{T}$ are $\{x_k\}_{k=0}^{LN-1}$.

$$x_k = x(k\frac{T}{LN}) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i \, e^{\frac{j2\pi ki}{LN}}, k \in [0, LN-1] \tag{2}$$

Where $L$ is the oversampling ratio.

The PAPR of the OFDM signal is defined by [13]

$$PAPR = \frac{\max\limits_{0 \leq k \leq LN-1} |x_k|^2}{E[|x_k|^2]} \tag{3}$$

Where $E\{.\}$ denotes the mathematical expectation. To have a good approximation of the PAPR, the oversampling rate must be more than 4 i.e. $L \geq 4$. The PAPR of the OFDM signal is a random variable. To investigate the behavior of this variable, it is common to utilize the Complementary Cumulative Density Function (CCDF). The $CCDF(PAPR_0)$ shows the probability of that the value of PAPR is greater than $PAPR_0$ i.e.[13]

$$CCDF(PAPR_0) = pr\{PAPR > PAPR_0\} \tag{4}$$

### III. THE PROPOSED TECHNIQUE FOR PAPR REDUCTION

In this section, the system model of an OFDM system with the 16QAM constellation and a coding rate of $\frac{k}{n}$ is described. The input bits are partitioned into the vectors of the length $\frac{4Nk}{n}$ (4 is order of 16QAM modulation). The vector of the input bits, **B**, is partitioned into subblocks of the length $k$. The number of sub blocks is $M = \frac{4N}{n}$. i.e.

$$B = \begin{bmatrix} B_1^T & B_2^T & ..... & B_M^T \end{bmatrix}^T$$
$$B_i = \begin{bmatrix} b_{i,1} & b_{i,2} & ..... & b_{i,k} \end{bmatrix}^T \tag{5}$$

Where $b_{i,j}$ is the $j$th bit of the $i$th sub block. Each sub block is encoded by a linear block code, $C(n,k)$, with generator matrix of $G$, i.e.

$$C_i = B_i G \tag{6}$$

The vector $C_i$ (of the length $n$) is the $i$th coded sub block. It is assumed that the linear block code $C(n,k)$, can correct t bits per sub block. The encoded sub blocks, $C_i \ \ i = 1, ..., M$, are grouped into the block $C$, i.e.

$$C = \begin{bmatrix} C_1^T & C_2^T & ..... & C_M^T \end{bmatrix}^T$$
$$C_i = \begin{bmatrix} c_{i,1} & c_{i,2} & ..... & c_{i,n} \end{bmatrix}^T \tag{8}$$

Where $c_{i,j}$ is the $j$th bit of the $i$th coded sub block.

To generate the symbol vector $X$, the vector $C$ must be mapped to the constellation points. Every 4 bits of the vector **C** are mapped to a point of 16QAM constellation. As shown in Fig.1, two MSBs choose quadrant and two LSBs select one of the 4 points in the chosen quadrant [14]. At the receiver side, it is obvious that two LSBs are potentially more affected by the channel error and two MSBs have a lower error probability. In the proposed method, the power of error correction capability for MSB bits are used for PAPR reduction.
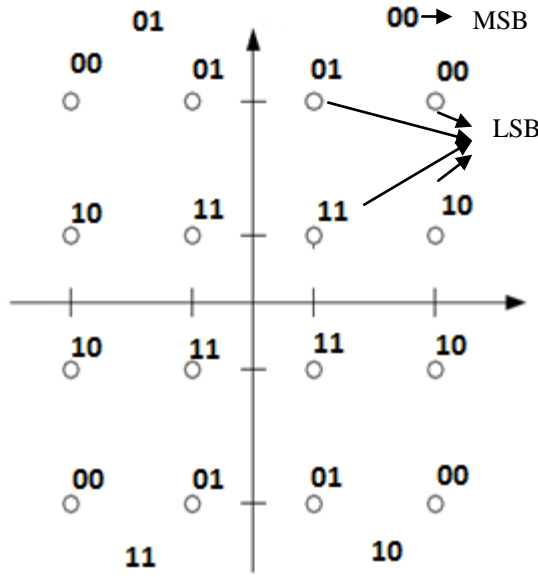
Fig. 1. Constellation points for 16-QAM modulation

It is proposed that the MSB bits are chosen from the even sub blocks and LSB bits from the odd sub blocks. If $Q(.)$ is defined as the mapping operation the symbol $X_{m,i}$ is generated by

$$X_{m,i} = Q(c_{2m-1,2i-1} \; c_{2m-1,2i} \; c_{2m,2i-1} \; c_{2m,2i})$$
$$i = 1, \dots, \frac{n}{2}, \quad m = 1, \dots, \frac{M}{2} \tag{9}$$

Every two sub blocks $\boldsymbol{C}_{2m-1}, \boldsymbol{C}_{2m}$ generate $\frac{n}{2}$ symbols, the vector $\boldsymbol{X}_m$ is defined by

$$\boldsymbol{X}_m = \left[ X_{m,1} \; X_{m,2} \dots X_{m,\frac{n}{2}} \right]^T \tag{10}$$

M/2 sub blocks $\boldsymbol{X}_m, m = 1, \dots, M/2$, are grouped into the final vector $\boldsymbol{X}$ as

$$X = \left[ X_1{}^T, X_2{}^T, \dots, X_{\frac{M}{2}}{}^T \right]^T \tag{11}$$

The OFDM signal $\boldsymbol{x}$ is generated from $\boldsymbol{X}$ using equation (2).

It's clear that the change in one bit of the even sub blocks $\boldsymbol{C}_{2m}$ changes a symbol strictly, because it changes the quadrants of the symbol. Also these bits have low error probability in channel, thus it is proper to add intentional error in the sub blocks $\boldsymbol{C}_{2m}$ to reduce the PAPR of OFDM signal. It is clear that the intentional error can be corrected at the receiver without any side information. In the proposed technique, it is assumed that the half of the capacity of error correction is used for PAPR reduction.

The error vector $\boldsymbol{E}$ is defined as the intentional error vector that is added to each encoded sub block $\boldsymbol{C}_{2m}$. For simplicity, it assumed that only one intentional error is added to each sub block. Thus, only one symbols is changed in each sub block $\boldsymbol{X}_m$. There are $n$ possible locations where the error bit

can be inserted into sub block. The position that generates the OFDM signal with the least PAPR is chosen. At the first step, the sub block **E** is added only to the first even encoded sub block i.e. $C_2$

$$C_2' = C_2 + E \tag{12}$$

   Thus one of the symbols of the sub block $\boldsymbol{X_1}$ is changed. By changing the error location the changed symbol is swept in the sub block. By comparison of the PAPR of OFDM signals, the best state of adding error with the lower PAPR is chosen. The best state is saved and $\mathbf{C_2}$ is replaced by $\mathbf{C_2'} = \mathbf{C_2} + \mathbf{E}$ . In the next step, the sub block $\boldsymbol{E}$ is added to the next encoded even sub block $\mathbf{C_4}$ and the best state for adding the error sub block $\boldsymbol{E}$ is found and $C_4$ is replaced by $C_4' = C_4 + E$ . This process is continued and the optimum OFDM signal with the least PAPR value is transmitted. In this method, IDFT must be computed $n$ times. Since the two consecutive OFDM vectors differ only in one symbol, the computations complexity can be reduced. If $\boldsymbol{x}$ is the original OFDM signal, then changing the symbols $X_i$ to $X_i'$ will change the samples of the time domain OFDM signal as follows:

$$x'_k = x_k + (X_i' - X_i)e^{\frac{j2\pi ki}{LN}} \tag{13}$$

   Thus, the calculation of the new OFDM signal with change of a single symbol needs $LN - 1$ Complex Multiplications (CM) and $LN$ Complex Additions (CA).

## IV.   SIMULATION RESULT

   The performance of proposed technique is evaluated by simulation. It is assumed that the input bit stream is divided into $M = 8$ sub blocks (for 64 subcarriers) and  $M = 16$ sub blocks (for 128 subcarriers) with 21 bits in each sub block. Each part is encoded by a BCH encoder with ( $n = 31, k = 21$). This code can correct two bits in each sub block. An un-coded bit is inserted at the end of each sub block. The performance of the proposed method has been compared with that of SLM method.

### *A.  PAPR reduction*

   For the computation of the CCDF of PAPR,  $10^4$ OFDM frames are generated. Oversampling ratio is assumed to be  $L = 4$. Figures 2 and 3 show the PAPR reduction performance of the proposed technique for OFDM signals with $N = 128$ and 64 symbols. The performance of the proposed method has been compared with that of SLM method with $U = 16$ and un-coded OFDM. In SLM method $U$ different representations of data symbols are generated and the representation with the least PAPR is selected and transmitted. The representations are generated by the multiplication of the data symbols by $\boldsymbol{U}$ different predefined vectors of +/-1. The index of the selected rotating vector must be transmitted to the receiver as the side information.
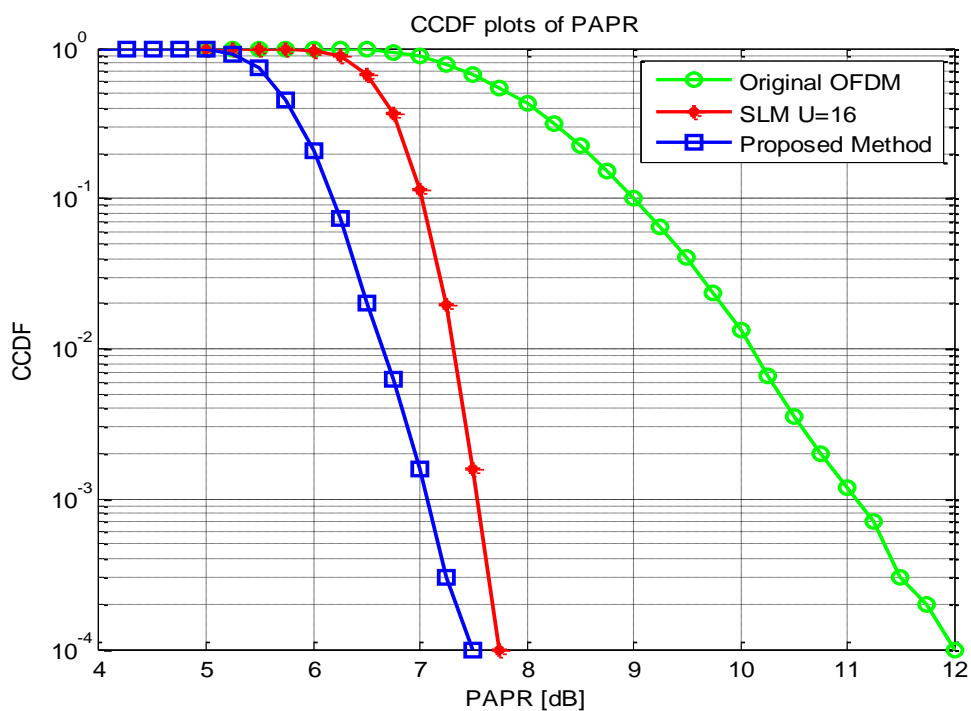
Fig. 2. CCDF of the PAPR for OFDM signal with 16-QAM-modulation ($N = \mathbf{128}$)
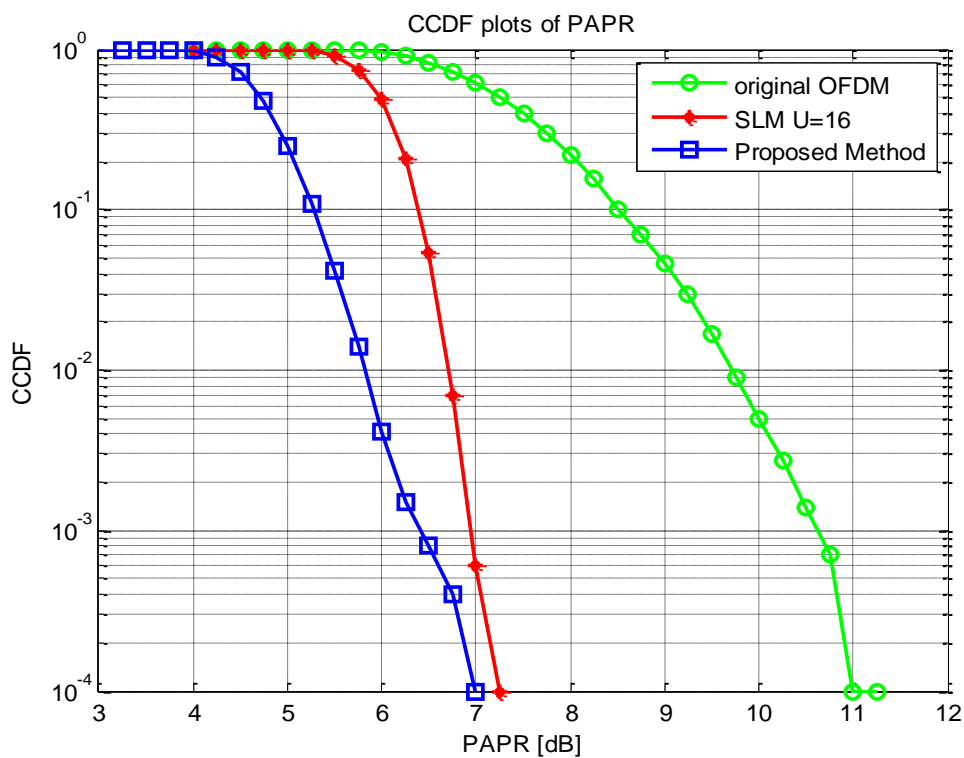


Fig. 3. CCDF of the PAPR for OFDM signal with 16-QAM-modulation ($N = \mathbf{64}$)

As shown in Fig.2 and 3 the proposed method has a PAPR reduction of about 4.5 dB for OFDM with $N = 128$ and 4 dB for OFDM with $N = 64$ while these values for the SLM method are 4.25 dB and 3.75 dB, respectively. Unlike the SLM method, the proposed method does not need SI.

## B. BER performance

The high PAPR makes distortion in the OFDM signal because of the nonlinear performance of HPA. The amplitude distortion of Solid State Power Amplifier (SSPA) can be modeled as [15]:

$$f\big(A(t)\big) = \frac{A(t)}{\left[1 + (\frac{A(t)}{A_{sat}})^{2p}\right]^{\frac{1}{2p}}} \tag{14}$$

$A(t)$ and $f(A(t))$ are the amplitudes of the input and output OFDM signals, respectively and $A_{sat}$ is the output saturation amplitude. The Input backoff (IBO) is defined by

$$IBO = 10log_{10}(\frac{A_{sat}^2}{E\{|x(t)|^2\}}) \tag{15}$$

The parameter $p$ controls the smoothness of the transition from the linear region to the saturation region [15].

After passing OFDM signal through the SSPA, it is passed through a multipath channel with 5 paths with the average powers of [0 -8 -17 -21 -25] dB. The delay of the paths is assumed to be [0 3 5 7 9] samples. At the receiver, the symbols are hardly decided after the channel equalization and the channel decoding is applied to the blocks of the output bits of demodulator.

Figures 4 and 5 show the BER performance of the proposed method for 128 subcarriers with $IBO = 2\ dB$ and $3\ dB$ respectively. Also fig.6 and 7 show the BER performance of the proposed method for 64 subcarriers with $IBO = 2\ dB$ and $3\ dB$ respectively. As can be seen form these figures the proposed method improves the BER performance in comparison to the case that the coded OFDM system is transmitted without PAPR reduction. In these figures also the BER has been plotted for the case that the coded OFDM is transmitted without amplifier distortion. This curve is the upper bound of the PAPR reduction algorithm. As can be seen in the Fig. 4 and the Fig.6, the distances of the BER performance of the proposed method and the upper bounds are 4.5 dB and 6 dB for 128 and 64 subcarriers while these values are more than 15 dB for the case that PAPR reduction algorithm is not applied. Also in the Fig.5 and Fig.7 this distances are 2.5 dB and 3 dB for 128 and 64 subcarriers when the SSPA is used with $IBO = 3\ dB$.
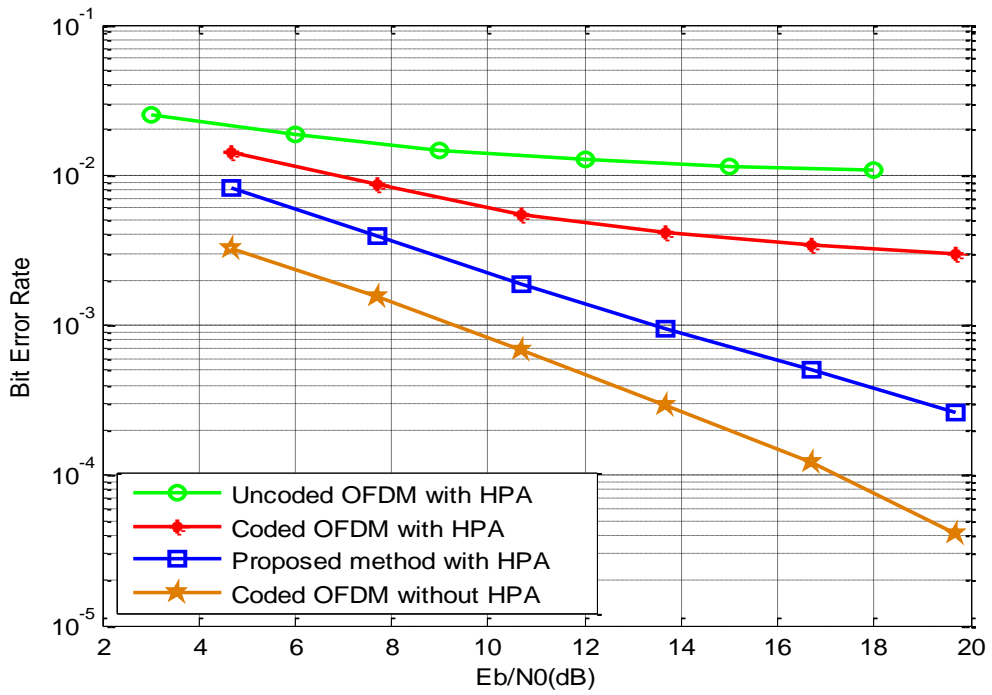
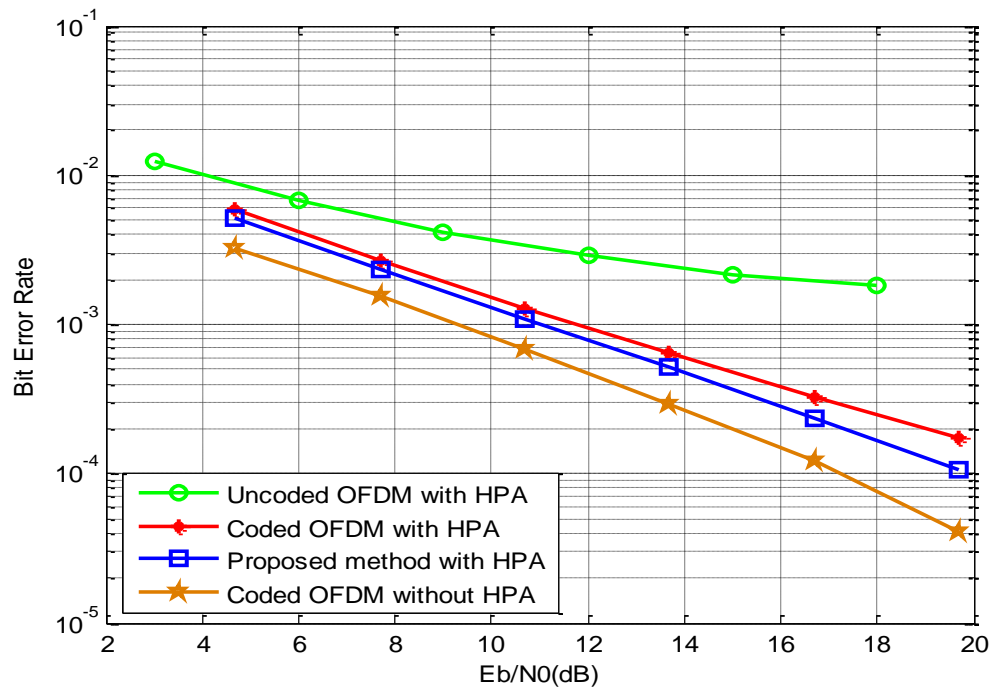Fig. 4. BER performance of PAPR reduction method in fading channel, 16QAM, **N = 128**, IBO=2 dB.



Fig. 5. BER performance of PAPR reduction method in fading channel, 16QAM, **N = 128**, IBO=3 dB.
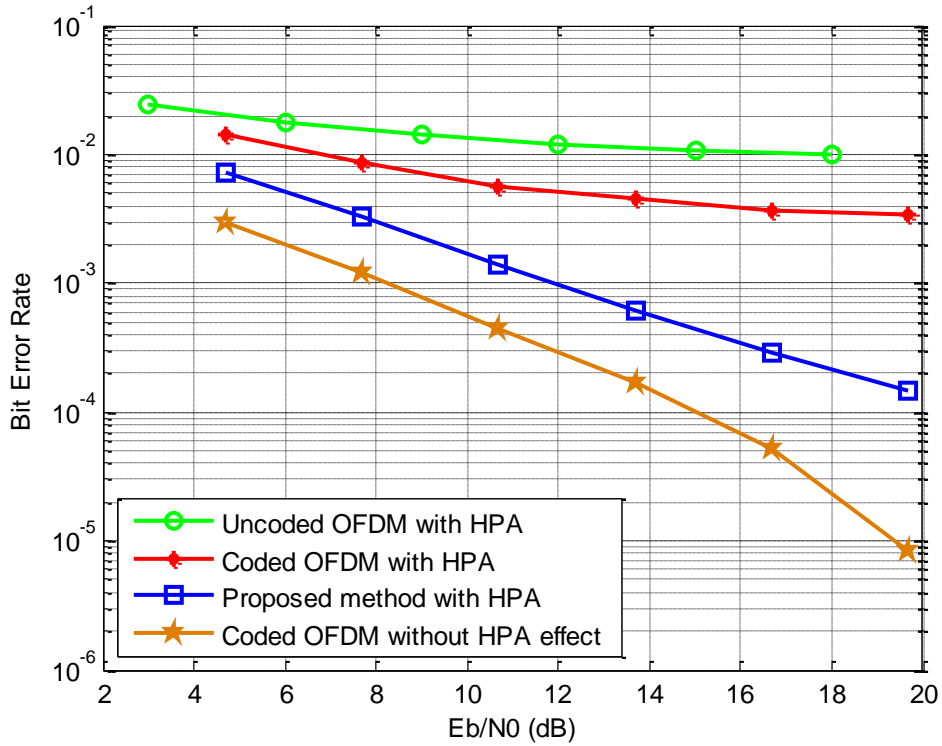
Fig. 6. BER performance of PAPR reduction method in fading channel, 16QAM, **N = 64**, IBO=2 dB.


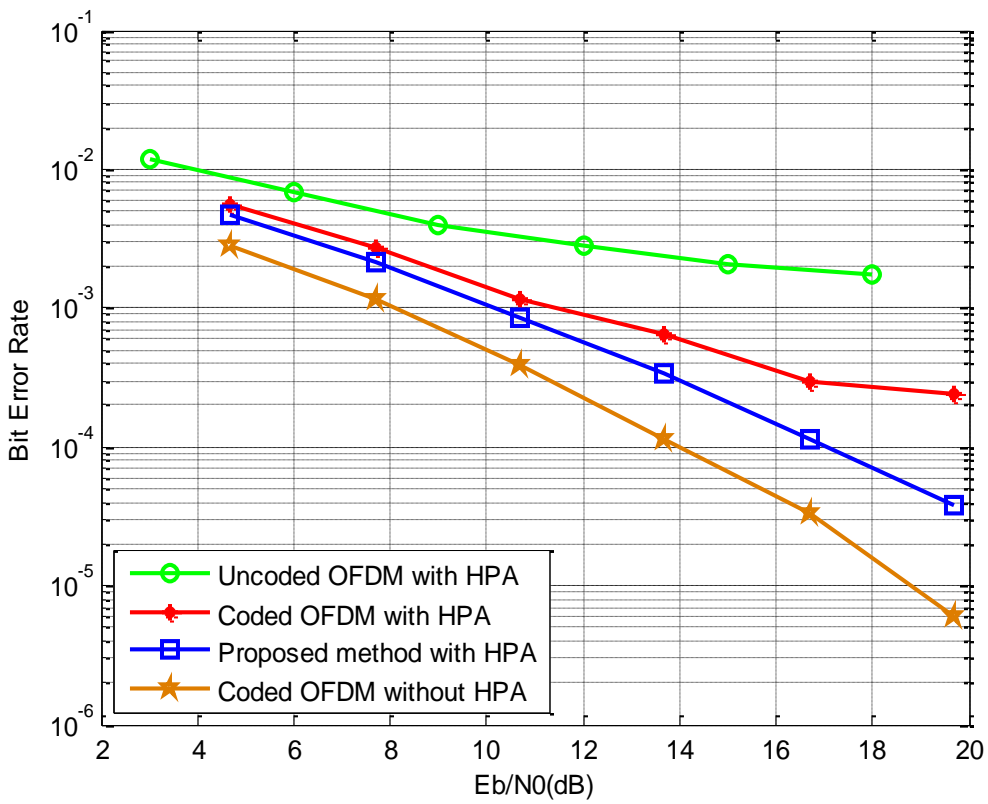
Fig. 7. BER performance of PAPR reduction method in fading channel, 16QAM, **N = 64**, IBO=3 dB.

V.  CONCLUSION

This paper proposed a coding for PAPR reduction of OFDM signals. In this method, the bits at the output of channel encoder are separated into MSBs and LSBs. The mapping to the constellation points is done such that the MSBs are less affected by the channel error. The capacity of the error correction at the MSBs is used for PAPR reduction. Simulation results show that the proposed method has a very close performance to that of the well-known SLM method while the main advantage of the proposed method is that it does not need side information (Unlike SLM method). The BER performance of the proposed method in the presence of the amplifier distortion was evaluated and it was shown that it decreases the effect of nonlinear distortion form more than 15 dB to about 5 dB.

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