

Improvement of the Reflection Coefficient of Waveguide-Fed Phased-Array Antenna Using Liquid Crystal

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Abstract— This work investigates the improvement of the active reflection coefficient of waveguide-fed phased-array antenna using liquid crystal layers. The anisotropic properties of liquid crystal layer can be employed to eliminate blind scan angle and improve the wide angle impedance matching of the waveguide-fed phased array antenna. The modal analysis of the waveguide-fed phased-array antenna which is covered with uniaxial anisotropic liquid crystal layers is considered. The analytical method is assumed for infinite arrays and is used to optimize the wide angle impedance matching over broad angular range of the array. The analytical results have been verified using simulation software. In this paper it is proposed to use anisotropic liquid crystal to improve the reflection coefficient of waveguide-fed phased-array antennas. Also it is proposed to employ the reconfigurable properties of liquid crystal to engineering the analytical optimization.

Index Terms— modal analysis, waveguide-fed phased-array antenna, liquid crystal.

I. INTRODUCTION

Liquid crystals are dielectric materials with anisotropic properties [1]. The nematic state is the most popular state of liquid crystals at microwave and millimeter-wave frequencies which is determined by a strongly anisotropic permittivity tensor [2]. The potential of liquid crystals as reconfigurable material come from their capability for continuous tuning with low power consumption, and their ability to integrate with printed circuit technologies [1-5]. In electromagnetic devices, which ordinary dielectrics are used, using optimized liquid crystal dielectrics can improve engineering designs; thus, resulting in improved system performance [1] and [6, 7].

The waveguide-fed phased-array antenna are periodic arrangement of waveguides which are affixed to apertures in a large perfect electric conductor (PEC) plane [8]. The apertures having exactly the same dimension as the waveguide cross-section. For wide angle scanning phased array antennas the magnitude of reflection change substantially with scan angle and there is a large variation of the

reflection over the range of scan angle [9-12]. Conventional matching techniques in the single element cannot compensate such reflection variation [13]. Wide angle impedance matching (WAIM) is a technique to reduce the variation of reflection coefficient with scan angle [13]. The WAIM method utilizes one or several dielectric matching layers in front of the array plane. One can change the parameters (such as the constitutive parameters values and thicknesses of one or more WAIM layers) of the matching structure to engineer the wide angular matching and achieve an acceptable reflection coefficient. Using uniaxial anisotropic sheet as a matching layer can improve the magnitude of reflection coefficient significantly. Sajuyigbe had proposed metamaterials as uniaxial anisotropic layer to improve the reflection coefficient of the phased array antennas. Besides using ELC metamaterials the WAIM of the proposed structure had improved up to 50 degree in azimuthal planes [14].

In this communication it is proposed to use liquid crystal as uniaxial anisotropic matching layer to improve the reflection coefficient of the waveguide-fed phased array antennas. Liquid crystal structures in comparisons with the ELC metamaterial, are readily realizable and the loss tangent is low rather than ELC metamaterial cells. In this paper realizable WAIM is improved up to 70 degree in azimuthal E- plane and H-plane. In this work, we attempt to analyze the waveguide-fed phased-array antenna, which is covered with liquid crystal dielectric layers. As well we have proposed to use reconfigurable properties of liquid crystals in order to optimize the reflection coefficient from the aperture plane of waveguide-fed phased-array antennas. To best knowledge of the authors it is the first time, which liquid crystals are used to engineer the matching range of the phased array antennas.

II. LIQUID CRYSTAL BEHAVIOR

Liquid crystals are dielectric materials with anisotropic properties, which are classified into three distinct meso-phases between solid/crystalline and liquid/isotropic phases [1]. This meso-phases are categorized by orientation and positional order into nematic, smectic, and cholesteric [2]. The nematic state is the most popular state of liquid crystals at microwave and millimeter-wave frequencies which is determined by a strongly anisotropic permittivity tensor. In this phase the average orientation of the molecular axes is characterized by the director, which is defined as the long axis direction of the rod-shaped liquid crystal molecules. Noticeable that the director denoted here as \vec{n} .

The anisotropy tensor represents a relative permittivity ϵ_{\parallel} parallel to the director direction \vec{n} and ϵ_{\perp} perpendicular to the director direction \vec{n} [3-5]. In a capacitive structure, where a liquid crystal cell is sandwiched between two metal electrodes, the tensor can be engineered using initial alignment of the liquid crystal molecules [6, 7] and also applying a bias voltage [1].

If the bias voltage equals to zero ($V_b = 0V$), the director \vec{n} will be parallel to the metallic layers. Therefore the relative permittivity tensor relevant for the interlayer material is defined as

$$\vec{\varepsilon}_{\perp} = \begin{bmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{bmatrix}. \quad (1)$$

For this state, the scalar effective relative permittivity can be estimated as $\varepsilon_{eff} = \vec{\varepsilon}_{\perp}(zz) = \varepsilon_{\perp}$. This is often referred to as the perpendicular state [1]. As the bias voltage V_b begins to enhance ($V_{th} \leq V_b \leq V_{max}$) the orientation of the director \vec{n} continuously varies from the perpendicular to parallel state.

The discrepancy between the perpendicular and parallel state is defined as the dielectric anisotropy $\Delta\varepsilon$ of the uniaxial liquid crystal molecules and can be represented as

$$\Delta\varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp}. \quad (2)$$

The general permittivity tensor for intermediate state may be represented as [1]

$$\vec{\varepsilon}_i = \begin{bmatrix} \varepsilon_{\perp} + \Delta\varepsilon \sin^2\theta & 0 & \Delta\varepsilon \sin\theta \cos\theta \\ 0 & \varepsilon_{\perp} & 0 \\ \Delta\varepsilon \sin\theta \cos\theta & 0 & \varepsilon_{\perp} + \Delta\varepsilon \cos^2\theta \end{bmatrix}, \quad (3)$$

Where in the above equation θ is the average angle among the molecular axis and the director \vec{n} .

Finally for V_b equal or greater than V_{max} , it becomes

$$\vec{\varepsilon}_{\parallel} = \begin{bmatrix} \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{bmatrix}. \quad (4)$$

III. PHASED-ARRAY ANTENNA AND MODAL ANALYSIS OF PERIODIC PLANAR PHASED-ARRAY OF ANTENNAS

The phased-array antenna is defined as a periodic arrangement of antennas that being used to achieve a strongly directional radiation whose scan direction may be varied using the phase gradient applied across the array [14]. A usual phased-array structure formed of open-ended waveguides in which each waveguide is attached onto apertures in a large perfect electric conductor (PEC) plane with apertures having just the same dimensions as the open-end of the waveguide.

To optimize the active reflection coefficient, active element admittance, the ratio of radiated power to input power be maximized at all scan angles of the phased-array structure; for an infinite array this ratio, in the far field zone, is approximated as [14-16].

$$\frac{P(\theta, \varphi)}{P_i} = (1 - |\Gamma(\theta, \varphi)|^2) f(\theta). \quad (5)$$

In the above $f(\theta)$ shows the physical limitation and changes according to geometric consideration [14, 17]. The output transmitted power is dependent on the scattering parameters, $S_{11}(\theta, \varphi)$ (or $\Gamma(\theta, \varphi)$) and as well on the geometric consideration, such as effective aperture section

which explicitly is included in $f(\theta)$. Note that $\Gamma(\theta, \varphi)$ being the reflection coefficient at the aperture interface of the array structure. The changing of the effective aperture size is a physical limitation, thus it is particularly important to minimize $\Gamma(\theta)$ over a very wide range of angles, especially at large angles. One may use uniaxial anisotropic liquid crystal layers above the array plane and select constitutive parameters to minimize reflection coefficient over a very wide scan range.

In a real array the elements interact with each other and alter the element admittances in comparison with the isolated elements. This interaction is known as mutual coupling. Mutual coupling varies the amplitude excitation and the phase of each element and result in the total array pattern different from isolated case. Also these observations depend on frequency and scan direction. Thus, for exact analysis the pattern of the array must include these effects in the amplitude and the phase of excitation. Therefore the pattern of each element acting under influence of coupling effects. As a result by changing the scan angle the phasing relationship between any two adjacent waveguide elements varies and changes mutual coupling, thus subsequently the active element admittance will change.

It is known that the problem of calculating the active element admittance of an element by neglecting the edge of a large planar phased-array is significantly simplified if the array is assumed to have infinite dimensions. Therefore the problem is reduced to the study of the transmission of the electromagnetic energy between the periodic waveguide-fed elements and the periodic radiation half space, i.e., the problem becomes a waveguide discontinuity problem [10, 18].

The goal of this section is to establish a very general systematic method for the analysis of waveguide-fed phased-array antennas which is covered with finite uniaxial anisotropic liquid crystal layers. The lattice of the element is arbitrary, the radiators are rectangular or circular waveguide.

Several methods have been proposed to derive the active element admittances of phased-array [10, 15-19]. Here we used the method proposed by Borgiotti to find the active element admittance [14]. The method is accurate and can be extended to more general configuration. The method is applicable to array with waveguide elements for which the Fourier transform of the fields distribution are known.

The coefficient of the waveguide modal expansion in the waveguide-fed and also the coefficient of tow-dimensional Floquet series in the radiation half-space are determined by approximately satisfying the boundary condition on the array interface.

If all the unknown of no direct interest (the amplitude of the waveguide modes and space harmonics) are removed from the calculations, as a simple final result, the active element admittance of an element is obtained as the ratio of tow determinants of orders N and $N-1$ where N is the number of waveguide modes considered. The elements of the determinants are series similar to the grating-lobe series [18].

Using the method proposed in [18] for simplification in presentation of equations we, define the following notation

$$Y_{ki}(k_{to}) = \frac{4\pi^2}{c} \sum_{k_{topq}} \begin{bmatrix} \varepsilon_{k\rho}^*(k_{topq}) \varepsilon_{i\rho}(k_{topq}) Y_{TM}(|k_{topq}|) \\ -\varepsilon_{k\psi}^*(k_{topq}) \varepsilon_{i\psi}(k_{topq}) Y_{TE}(|k_{topq}|) \end{bmatrix} \quad (6)$$

Where

$$k_{to} = \hat{x}k_x + \hat{y}k_y, \quad (7)$$

$$k_{topq} = k_{to} + pt_1 + qt_2, \quad (8)$$

$$k_x = k_o \sin \theta \cos \varphi, \quad (9)$$

$$k_y = k_o \sin \theta \sin \varphi. \quad (10)$$

In the above equations, k_{topq} indicates the lattice of grating lobe in the visible and invisible space over the array aperture and $Y_k(k_{to})$ indicates the active element admittance, driving point admittance, at the scan angle k_{to} . The two vectors t_1 and t_2 indicate the periodicity of the array lattice in the reciprocal space, whereas the two arbitrary scalars p and q represent the contribution of evanescent or propagating plane waves in space, exactly above the open-end of the waveguide-fed. In equation (6) the star denoting the complex conjugate and C denoting the basic lattice cell area. The terms of admittances with $i = k$ are known as the grating-lobe series of an element whose transverse field distribution is given by $e_i(t)$.

In the above equation

$$\varepsilon_i(k_t) = \frac{1}{2\pi} \iint e_i(t) e^{jk_t t} dt, \quad (11)$$

Where the integral is taken over the entire cross section of waveguide aperture. We can write [20]

$$\varepsilon_i(k_t) = \varepsilon_{i\rho}(k_t) \hat{\rho} + \varepsilon_{i\psi}(k_t) \hat{\psi}. \quad (12)$$

In the above equations the $\hat{\rho}$ and $\hat{\psi}$ are unit vectors in the radial and circumferential directions. Using this decomposition leading to simplify the calculation of $Y(k_{to})$, since the first and second parts of (12) indicate respectively, the TM and TE terms of a field whose tangential component on the aperture plane is $e_i(t)$. Subsequently, the tow constant of proportionality $Y_{TM}(|k_t|)$ and $Y_{TE}(|k_t|)$, the surface admittances of TM and TE fields, which depend only upon the k_t , associate $\varepsilon_{i\rho}(k_t)$ and $\varepsilon_{i\psi}(k_t)$ respectively to ψ and ρ components of the Fourier transforms of the tangential magnetic field [21]. These admittances can be derived easily for infinite half free space over the interface above the phased-array [22, 23].

For perpendicular state ($V_b = 0V$), using Maxwell's equations, the admittances parameters Y_{TE} and Y_{TM} for infinitely thick uniaxial anisotropic liquid crystal matching layer placed directly above the aperture-air discontinuity, are derived.

$$Y_{TE}(|k_{topq}|) = -\frac{H_x}{E_y} = \frac{k_{z,TE}}{\omega\mu_0\mu_{\parallel}} \quad (13)$$

$$k_{z,TE} = \sqrt{\omega^2\mu_0\mu_{\parallel}\varepsilon_0\varepsilon_{\perp} - k_t^2\frac{\mu_{\parallel}}{\mu_{\perp}}} \quad (14)$$

$$Y_{TM}(|k_{topq}|) = \frac{H_y}{E_x} = \frac{\omega\varepsilon_0\varepsilon_{\parallel}}{k_{z,TM}} \quad (15)$$

$$k_{z,TM} = \sqrt{\omega^2\varepsilon_0\varepsilon_{\parallel}\mu_0\mu_{\perp} - k_t^2\frac{\varepsilon_{\parallel}}{\varepsilon_{\perp}}} \quad (16)$$

Also for parallel state ($V_b \geq V_{max}$) these parameters are given in the following.

$$Y_{TE}(|k_{topq}|) = -\frac{H_x}{E_y} = \frac{k_{z,TE}}{\omega\mu_0\mu_{\perp}} \quad (17)$$

$$k_{z,TE} = \sqrt{\omega^2\mu_0\mu_{\perp}\varepsilon_0\varepsilon_{\perp} - k_t^2\frac{\mu_{\perp}}{\mu_{\parallel}}} \quad (18)$$

$$Y_{TM}(|k_{topq}|) = \frac{H_y}{E_x} = \frac{\omega\varepsilon_0\varepsilon_{\perp}}{k_{z,TM}} \quad (19)$$

$$k_{z,TM} = \sqrt{\omega^2\varepsilon_0\varepsilon_{\perp}\mu_0\mu_{\perp} - k_t^2\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}}} \quad (20)$$

For L-layers of finite thicknesses, the transmission line impedance transformation equation can be used [24]. The admittances can be computed using the following recursive procedure.

$$\left\{ \begin{array}{l} Y_{TE0} = -\frac{k_z}{\mu k} \\ Y_{TE} = Y_{TEl} \left[\frac{Y_{TE(l-1)} + Y_{TEl} \tanh(k_{z,TEl}d_l)}{Y_{TE(l)} + Y_{TE(l-1)} \tanh(k_{z,TEl}d_l)} \right] \end{array} \right. \quad l = 0,1,2, \dots, L-1 \quad (21)$$

$$\left\{ \begin{array}{l} Y_{TM0} = \frac{k}{\mu k_z} \\ Y_{TM} = Y_{TMl} \left[\frac{Y_{TM(l-1)} + Y_{TMl} \tanh(k_{z,TMl}d_l)}{Y_{TM(l)} + Y_{TM(l-1)} \tanh(k_{z,TMl}d_l)} \right] \end{array} \right. \quad l = 0,1,2, \dots, L-1 \quad (22)$$

Where “ d_l ” is thickness of the finite anisotropic layer and the indice l represent the different layer, k is the free space wavenumber and k_z is the plane wave propagation constant in free space. For visible space:

$$k_z = \sqrt{k^2 - k_t^2}.$$

From the continuity of the electric and magnetic field on the interface plane of the array, following set of equations can be derived [18]

$$V_o Y(k_{to}) - \sum_{i=0}^{N-1} V_i Y_{oi}(k_{to}) = 0; \tag{23}$$

$$V_k Y_k + \sum_{i=0}^{N-1} V_i Y_{ki}(k_{to}) = 0. \tag{24}$$

To solve equations simultaneously equations (23) and (24), the following Nth order determinant must be equal to zero.

$$\begin{vmatrix} Y(k_{to}) - Y_{0,0} & -Y_{0,1} & \dots & -Y_{0,N-1} \\ -Y_{1,0} & -Y_1 - Y_{11} & \dots & -Y_{1,N-1} \\ \dots & \dots & \dots & \dots \\ -Y_{N-1,0} & -Y_{N-1,1} & \dots & -Y_{N-1,N-1} \end{vmatrix} = 0 \tag{25}$$

From this expression the active element admittance can be obtained as

$$Y(k_{to}) = \frac{\begin{vmatrix} Y_{0,0} & Y_{0,1} & \dots & Y_{0,N-1} \\ Y_{1,0} & Y_1 + Y_{11} & \dots & Y_{1,N-1} \\ \dots & \dots & \dots & \dots \\ Y_{N-1,0} & Y_{N-1,1} & \dots & Y_{N-1,N-1} + Y_{N-1} \end{vmatrix}}{\begin{vmatrix} Y_{1,1} + Y_1 & Y_{1,2} & \dots & Y_{1,N-1} \\ Y_{2,1} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Y_{N-1,1} & Y_{N-1,2} & \dots & Y_{N-1,N-1} + Y_{N-1} \end{vmatrix}} \tag{26}$$

In the above equations, Y_{ki} is called the cross polarization admittance between different modes in the waveguide element, whereas Y_k in (26) which is a function of frequency, not the scan angle, indicates the modal admittances of waveguide mode k [14, 18, 24]. Indices i and k represent the contributing of waveguide modes from the dominant propagating mode at $i = 0$ or $k = 0$ to the slightest contributive non-propagating mode at $i = \infty$ or $k = \infty$. The summation in the equation (23) and equation (24) are truncated at a finite sentences $N - 1$. Because the higher order modes far away from the dominant mode have negligible contribution on the summations. It is shown that if $i = k = 0$ then $Y_{0,0}$ represent a special case where only the dominant waveguide mode is being considered, same as the equation (12) in [21]. In this special case the admittances equation is known as the grating lobe series.

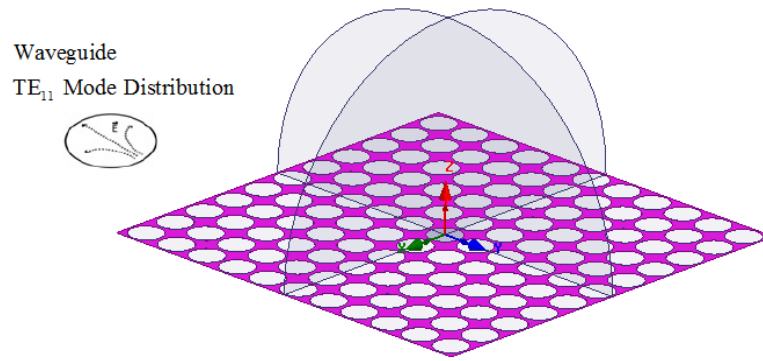


Fig. 1. Array configuration

Equation (26) is the desired result. If the Fourier transforms of the vector mode functions are known, a simple Matlab script may evaluate the active element admittance equation. Fourier transforms of the vector mode functions for rectangular and circular waveguides are given in closed form, thus for rectangular and circular waveguides the desired equation readily is applicable.

IV. RESULTS AND DISCUSSION

The active element admittances for the waveguide-fed phased array antenna as a function of scan angle is analyzed. The aperture plane of the array is covered with a typical liquid crystal, GT3-23001.

Analytically results are verified using simulation software CST and furthermore finite element method solver Ansoft HFSS. Array configuration is shown in Fig. 1. Two typical arbitrary grids, square and hexagonal, is considered as shown in Figs. 2 and 3. The elements of the arrays are circular waveguides, which are loaded with $\epsilon_r = 2.1$ and $\epsilon_r = 2.54$ dielectrics respectively for square and hexagonal lattice. The arbitrary lattice dimensions are equal to 9 mm and 10.5 mm respectively for square and hexagonal lattice. The circular waveguides are affixed to an aperture on a PEC plane. The thickness of the matching layer is 1.65 mm for hexagonal lattice. Also the matching layer offsets 0.65 mm from the aperture plane. Of note, thickness of the matching layer is 1.3 mm for square lattice. Each element in the phased-array is cross-fed to excite two dominant TE_{11} mode (in this work the dominant TE_{11} mode is considered as the reference mode).

Using analytical method according to equation (25), the active element admittance in H-plane versus different scan angles is plotted and compared with the HFSS and CST simulation results is plotted in Fig. 4. This figure is corresponded to the hexagonal lattice. In the proposed structure, an anisotropic matching layer (GT3-23001) is placed over the aperture face. As it can be seen in Fig. 4, the reflection coefficient is below -10 dB up to scan angle equal to 70 degree. The comparisons between analytical and simulation results show the accuracy of the analytical solution. It is more common to draw the transmitted power versus scan angle. The transmitted power, equivalent to reflection coefficient Fig. 4, is depicted in Fig. 5. In Fig. 5, equivalent to Fig. 4, the transmitted power

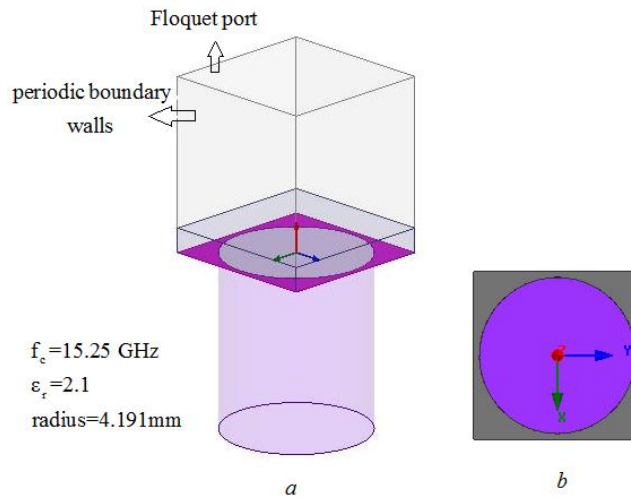


Fig. 2. Square unit cell of the phased-array.

- (a) Infinite array using master and slave boundary conditions and Floquet port used in HFSS simulations.
- (b) Top view of the unit lattice.

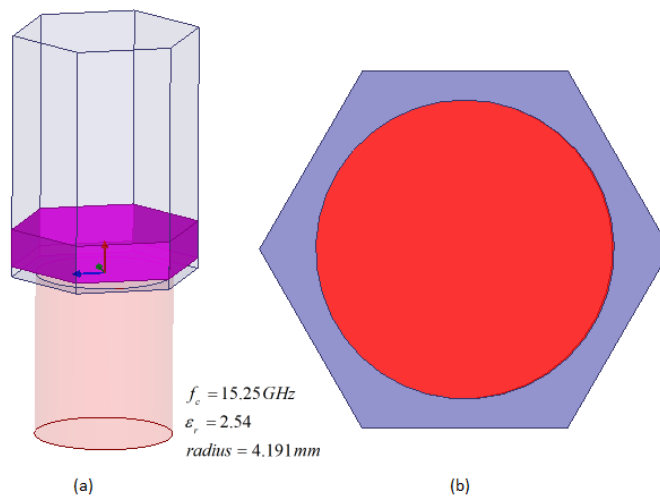


Fig. 3. hexagonal unit cell of the phased-array.

- (a) Infinite array. (b) Top view of the unit lattice.

is above 90 percent of the total input power up to 70 degree.

The results are extracted for the mentioned hexagonal and square lattice. To approximate the behavior of a phased-array system, it is common to look at the H-, D- and E-planes [$\varphi = 0^\circ$, $\varphi = 45^\circ$ and $\varphi = 90^\circ$ respectively]. As shown in Fig. 5, the matching range in H-plane is improved to 70 degree for hexagonal structure. As well as Fig. 6 shows that the matching range in E-plane is improved to 75 degree. In other words as shown in Fig 6 the reflection coefficient is under -10 dB up to 75 degree. Fig. 7 and 8 show H-plane and E-plane of the square lattice respectively. As shown in

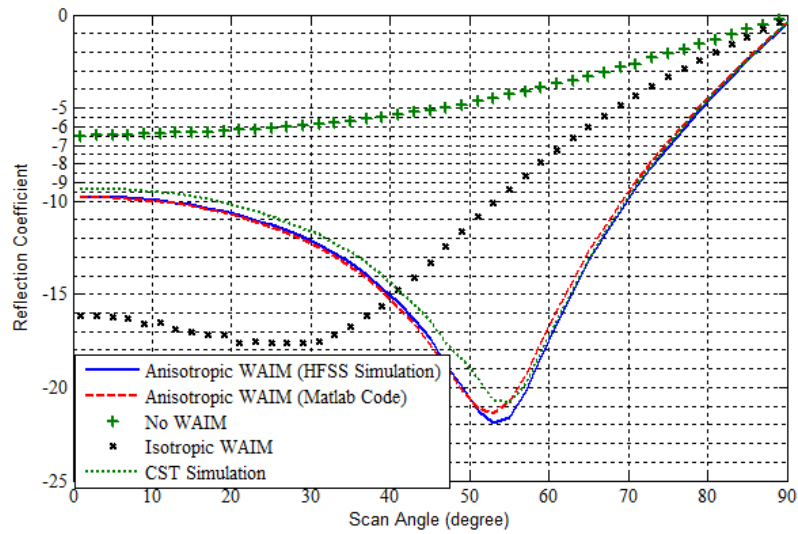


Fig. 4 H-plane of active reflection coefficient for the typical liquid crystal layer GT3-23001 with hexagonal lattice.

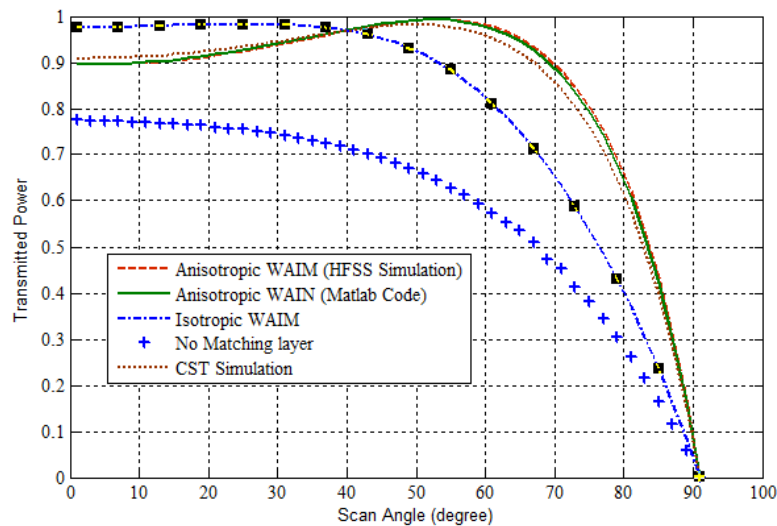


Fig. 5 H-plane of transmitted power for the typical liquid crystal layer GT3-23001 with hexagonal lattice.

Fig. 7, the matching range in H-plane is increased to 70 degree. Also Fig. 8 shows that the matching range in E-plane is improved to 70 degree. D-plane is a common inter-cardinal plane. It is noticeable our results show that, the matching range in D-plane is improved to 90 degree and 70 degree respectively for hexagonal and square structure (not depicted in this article).

These results are comparable with the result of the Sajuyigbe [14]. Sajuyigbe used ELC metamaterial layers and improved the wide angle impedance matching (WAIM) of waveguide-fed phased-array antenna up to 50 degree. In this paper using anisotropic liquid crystal, GT3-23001, WAIM is improved up to 70 degree.

It is known that the presence of dielectric layers on the perfect electric conductor plane leads to guided wave mode. The guided wave mode in this structure is the surface wave mode that is

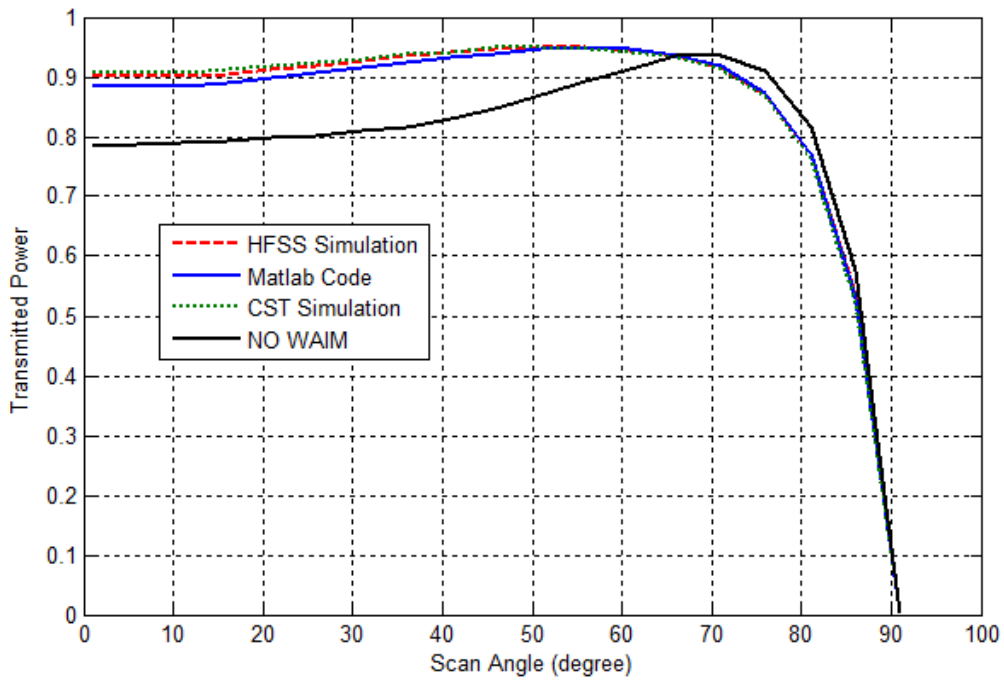


Fig. 6 E-plane of transmitted power for the typical liquid crystal layer GT3-23001 with hexagonal.

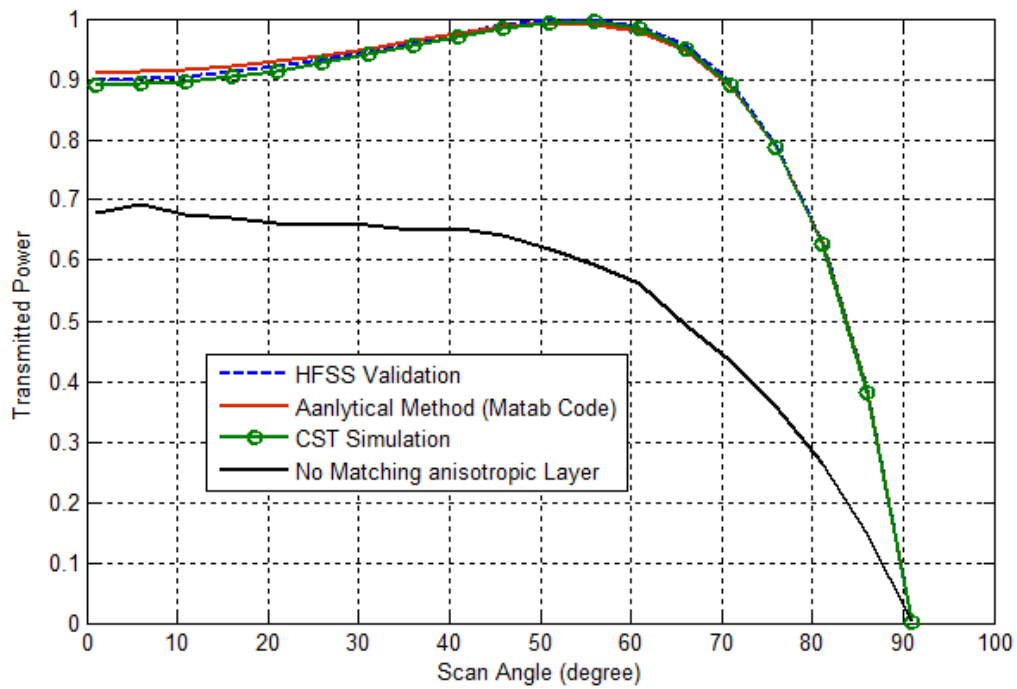


Fig. 7 H-plane of transmitted power for the typical liquid crystal layer GT3-23001 with square lattice.

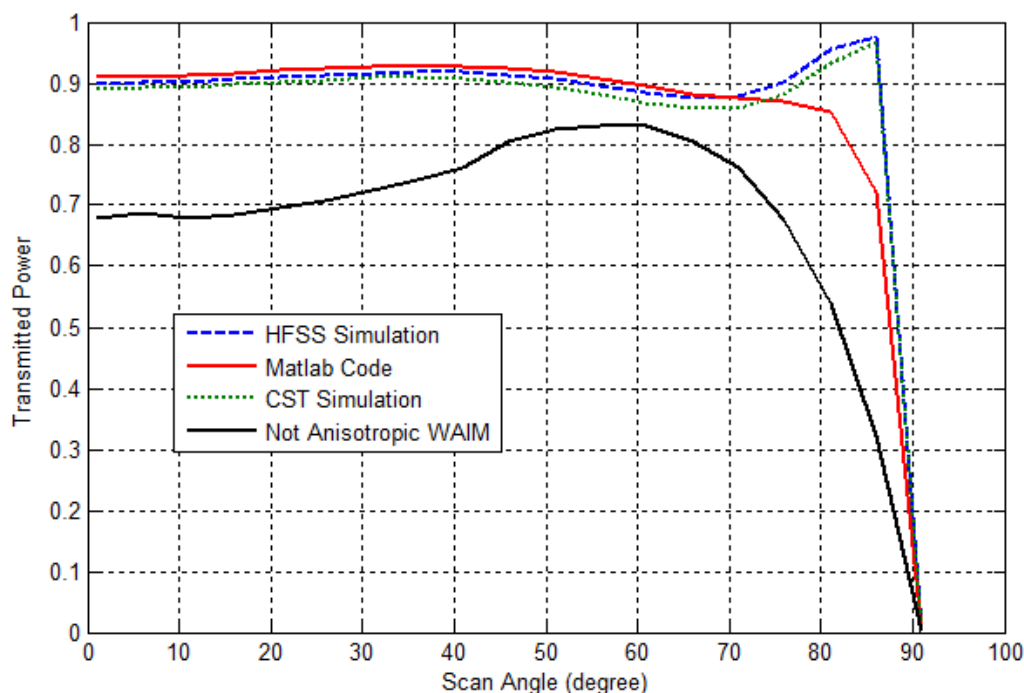


Fig. 8 E-plane of transmitted power for the typical liquid crystal layer GT3-23001 with square lattice.

supported by the matching anisotropic layer. If the propagation constant of a surface wave mode coincides with a Floquet mode, thus these two modes can couple strongly, leading to a resonance and scan blindness. In this paper, this kind of blindness spot could be controlled by changing parameters such as thickness of the matching layer and offsetting it from the aperture plane. Therefore, to eliminate blindness scan and improved the range of matching it is necessary to optimize these parameters. It is noticeable that the optimized parameters could obtained either analytically in mentioned method or using tuning tools of the simulations software [14]. Using analytical method optimized parameters could obtain much faster than simulations software.

V. CONCLUSION

In this work, a new type of anisotropic matching layer, liquid crystal, was demonstrated and applied to improve the scanning range of waveguide-fed phased-array antenna. The problem is demonstrated here using analytical method. Then using Ansoft HFSS and CST software, the obtained result are verified. The obtained results indicate that the proposed structure improved the wide angle impedance matching in the H-plane and E-plane up to 70 degree. It is suggested that using a bias voltage to engineering matching liquid crystal constitutive parameters (uniaxial ϵ , uniaxial μ), improvement of the wide angle impedance matching occurs.

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