Sum-Rate Maximization Based on Power Constraints for Cooperative AF Relay Networks

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Abstract- In this paper, our objective is maximizing total sum-rate subject to power constraints on total relay transmit power or individual relay powers, for amplify-and-forward single-antenna relay-based wireless communication networks. We derive a closed-form solution for the total power constraint optimization problem and show that the individual relay power constraints optimization problem is a quadratic programming which does not have a closed-form solution. To solve it, we propose a closed-form low-complex technique which introduces suboptimal solution as an upper bound of system sum-rate.

The performance of two new proposed algorithms are evaluated in the view of average achievable sum-rate and energy efficiency in different signal to noise ratios (SNRs). Considering the effect of the number of relays, the effect of channel gains of the source-relay as well as the relay-destination, and also the impact of the level of imperfect CSI are investigated. According to the simulation results, higher sum-rate can be achieved in total power constrained scenario for all SNRs but energy efficiency is approximately the same for both of them. In addition, higher sum-rate is achievable by increasing the number of relays and/or improving the quality of uplink and downlink channels as well as decreasing uncertainty of channels.

Index Terms- Cooperative, MIMO relay network, AF relay, Beamforming, Power control, Sum-rate.

I. INTRODUCTION

Providing wireless communications anytime anywhere needs bandwidth-efficient technologies. To improve the performance of these technologies, various diversity techniques in frequency, time, code, space or a combination of them may be applied. An interesting type of space diversity, namely cooperative diversity is considered in [1]–[3]. A three-node network is studied in [4], where the relay nodes between the sources and destinations help to have a non-direct reliable transmission by making virtual array antennas. This method, avoids the problems related to implementing antenna arrays such

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as, physical limitations, mutual coupling of array elements and so on. In addition, it makes the possibility of taking diversity and other advantages of multiple-input multiple-output (MIMO) systems and also applying two important ideas, namely power control and beamforming in MIMO systems.

It should be mentioned that for long distance communications between transmitter and receiver, the direct link between source and destination faces large attenuation. Hence, using relay-based networks, transceiver nodes can communicate with each other in two hops. In the first hop, transmitter sends signal to relay and relay makes a process on it. This process depends on the type of relay. Thereafter, in the second hop, the relay sends the processed signal to the receiver.

In terms of the type of processing, relays can be classified into different modes such as, amplify-and-forward (AF) [3], decode-and-forward (DF) [5], and compress-and-forward (CF) [6] which the simplest and popular one is AF but it is not as effective as DF and CF ones. AF relay linearly amplifies the signal and sends the amplified signal to the receiver. Unlike regenerative relays, AF relay has no impact on the noise because decoding is not performed. Although AF relays have no effect on decreasing error [7], AF relay proffers less complexity and lower delay associated with relay processing rather than the other schemes. Also, the AF relay has introduced a distributed space-time coding research area in relay-based networks [8]–[14].

The problem of resource allocation for different relaying schemes has been studied in [15]. In [16], joint beamforming and power control in receiver for MIMO relay networks are used to reduce co-channel interference (CCI). In [17], assuming the instantaneous Channel State Information (CSI) is known at relays and both end nodes, an AF strategy is investigated. Considering a predefined power constraint for each relay node, the relay nodes try to maximize the received signal to noise ratio (SNR), by adjusting their phase of the received signal and transmit powers according to the quality (gain) of channels. As shown in [17], by maximizing SNR at the receiver, some of the relay powers may be lower than their maximum allowable power. In usual wireless communication networks just one source and one destination communicate with each other through multi relays [18, 19, 20]. In the most of research works, there is no direct link between the source and the destination. In contrast, in some investigations such as [17], the direct link is also considered but this link is the dominant one. When the number of source-destination pairs is more than one, the proposed method in [18] is not valid. In [21, 22], the power consumption is minimized while the required signal to interference plus noise ratio (SINR) for each link is guaranteed. Authors use Semidefinite Relaxation (SDR) technique to change this problem to a Semidefinite Programming (SDP) problem which can be solved by using Interior Point (IP) methods. Total Leakage (TL) minimization algorithm which aims at minimizing the sum of powers of the interferers and noise from the relays was introduced in [23]. They maximized system sum-rate with total relay transmit power constraint for both AF and DF strategies. In [24], general MIMO relay network was considered. The authors of [24] proved that an inequality exists between sum-rate and total signal to total interference plus noise ratio (TSTINR). Instead of sum-rate, they maximized TSTINR and proposed a low complexity algorithm to optimize the users' encoders and decoders as well as relay beamforming matrices. Moreover, two algorithms were proposed in [25] to maximize total signal to total leakage ratio (TSTLR) by maximizing the numerator for the first approach and the difference between the numerator and the denominator for the second one. SDR technique was used to turn the problem into a SDP problem and MATLAB CVX toolbox was used to solve the problem.

Different relay-assisted problems in D2D communications and 5G systems were presented in [26]-[29]. In such cases, relays assist D2D users to have a high performance communication when the quality of direct link between D2D users are not good enough or D2D users are far from each other. The authors in [26] addressed an optimal relay selection algorithm in D2D transmission. Joint relay selection and power allocation problem under AF relaying protocol was proposed in [27] and a solution based on iterative Hungarian algorithm was proposed. Under DF relaying, [28] investigated the power allocation at the source and relay nodes to maximize the sum throughput of the system. They also formulated a joint power optimization problem for orthogonal frequency division multiplexing (OFDM) based transmission. Two low-complex energy efficiency-based optimization problems for the uplink of a cooperative multi-point 5G system were proposed in [29] which uses fractional programming methods. One of them maximizes the network global energy efficiency and the other one does it for the worst-case energy-efficient design.

Here, a network consisting of multi pairs of single-antenna source-destination that communicate with each other using multi non-regenerative single-antenna relays is considered. By considering the problem of distributed beamforming, it is assumed that the Second-Order Statistics (SOS) of the channel coefficients for both source-relay and relay-destination links are known. Two new optimization problems are proposed which maximize sum-rate subjecting to two types of relay's power constraints, totally and individually. It is shown that in total relays power constraint we can find a closed-form solution. We also show that for individual relay power constraints the problem has not a closed-form solution but by using semidefinite programming (SDP) and relaxation technique the problem is turned into a convex one, so it can be approximately solved by interior point methods. This optimization problem is solved by convex optimization (CVX) MATLAB toolbox. Also, in the case of individual relay power constraints, we recommend a closed-form and low-complex technique which can be used as the upper bound of system sum-rate. The proposed schemes are compared to [22], TL [23], [24] and TSTLR [25] schemes.

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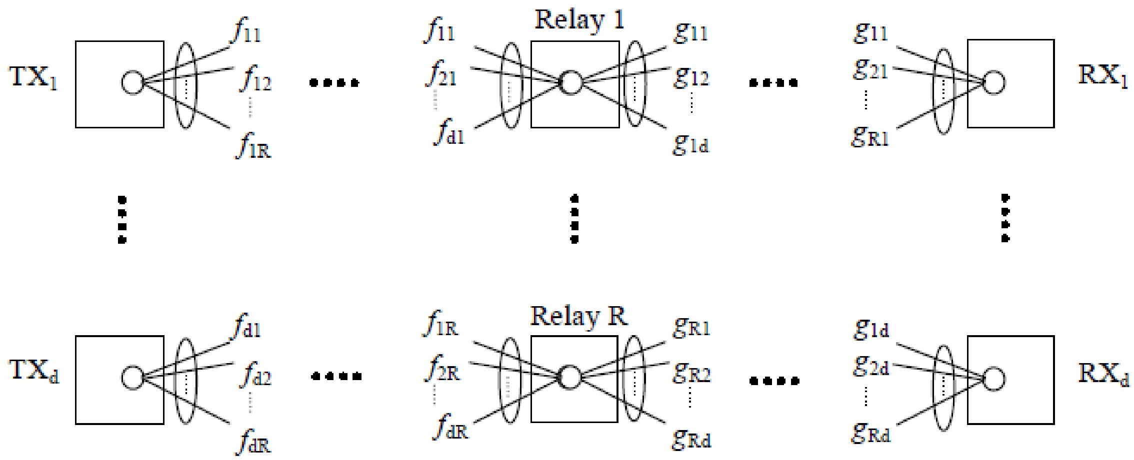

Fig 1. d source-destination system consisting of R relays

First, the performance of two new proposed algorithms are compared to that for algorithms of [22]-[25] in terms of average achievable sum-rate and energy efficiency in different noise powers. Considering the effect of the number of relays, the impact of channel gains (qualities) of the first hop as well as the second hop, and also the impact of the level of channel uncertainty are investigated.

The remainder of this paper is organized as follows. After illustrating the system model in Section 2, the maximization problems are formulated in Section 3, which present our proposed approaches with more details. Simulation results for the different scenarios of the system are presented in Section 4. Finally, conclusions and some suggestions for further research works are presented in Section 5.

II. SYSTEM MODEL

As shown in Figure 1, a system consisting of d source-destination pairs with R relays are considered that there is no direct link between the source and the destination. By weighting (adjusting the amplitude and phase) of received signal in each relay, relay multiplies its received signal by a weight and then sends it to the destination.

We assume that the coefficient matrix of the channel between the *p*th transmitter and the *r*th relay and also between the *r*th relay and the *p*th receiver are f_{rp} and g_{rp} respectively, the received signal at the *r*th relay can be formulated as (1) [22]

$$x_r = \sum_{p=1}^{d} f_{rp} s_p + v_r \tag{1}$$

where s_p is the transmitted symbol by the *p*th source and v_r is the zero-mean Additive White Gaussian Noise (AWGN) at the *r*th relay node.

Considering the following assumptions in this section, (1) can be reformulated as (2).

1- The noise in relay is AWGN, i.e.,$E\{v_r v_r^*\} = \sigma_v^2 \delta_{rr'}$, where σ_v^2 shows the noise power. $E\{.\}$ represents the statistical expectation, $(.)^*$ represents complex conjugate operator and $\delta_{rr'}$ denotes Kronecker's delta function.

2- The power of the *p*th source is p_p, i.e., $E\left\{\left|s_p\right|^2\right\} = p_p$.

3- The symbols for different sources are not correlated, i.e., $E\{s_p s_p^*\} = p_p \delta_{pq}$.

4- The symbols and the noise in *r*th relay are statistically independent.

$$\boldsymbol{x} = \sum_{p=1}^{d} \boldsymbol{f}_p s_p + \boldsymbol{v} \tag{2}$$

where $\boldsymbol{x} = [x_1\ x_2 \ldots x_R]^T, \boldsymbol{v} = [v_1\ v_2 \ldots v_R]^T$ and $\boldsymbol{f}_p = [f_{1p}\ f_{2p} \ldots f_{Rp}]^T$. Here, $(.)^T$ denotes the transpose operator.

The received signal in the *r*th relay will be multiplied by a complex coefficient w_r^*. Transmitted signal by relays are given as (3).

$$\boldsymbol{t} = \mathbf{W}^H \boldsymbol{x} \tag{3}$$

where $(.)^H$ represents Hermitian transpose operator, $\mathbf{W} = \mathrm{diag}([w_1, w_2, \ldots, w_R]^{\mathrm{T}})$ and $\boldsymbol{t}$ is an R × 1 vector.

Denoting $\boldsymbol{g}_k = [g_{1k}\ g_{2k} \ldots g_{Rk}]^T$ as the coefficients for channels between the relays and the *k*th destination, the received signal at *k*th receiver is given as (4).

$$\begin{aligned} y_k &= \boldsymbol{g}_k^T \boldsymbol{t} + n_k \\ &= \boldsymbol{g}_k^T \mathbf{W}^H \sum_{p=1}^{d} \boldsymbol{f}_p s_p + \boldsymbol{g}_k^T \mathbf{W}^H \boldsymbol{v} + n_k \\ &= \underbrace{\boldsymbol{g}_k^T \mathbf{W}^H \boldsymbol{f}_k s_k}_{\text{desired signal component}} + \underbrace{\boldsymbol{g}_k^T \mathbf{W}^H \sum_{p=1, p\neq k}^{d} \boldsymbol{f}_p s_p}_{\text{interference component}} + \underbrace{\boldsymbol{g}_k^T \mathbf{W}^H \boldsymbol{v} + n_k}_{\text{noise component}} \end{aligned} \tag{4}$$

where n_k is a Gaussian (normal) distributed noise at the *k*th receiver as $N(0, \sigma_n^2)$.

As noted in (4), the first term represents the desired information part of the received signal, the second one is the interference signals made by the other users, and finally two parts in the last term represent the noise. It shows that there are three unwanted signal groups at each destination:

1- Interference
2- Amplified noises from the relays to destinations
3- Receiver noise

In this research, it is supposed that all channel coefficients between sources and relays, the coefficients for the channels between relays and destinations, the transmitted signals, the noise signals in the relay and receiver are independent from each other. Table I summarizes the notations used in this paper as a quick reference.

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Table I. Description of symbols used in analytical modeling and simulations

Symbol	**Definition**
d	Number of source-destination pairs
R	Number of relays
f_{rp}	Coefficient matrix of the channel between the *p*th transmitter and the *r*th relay
g_{rp}	Coefficient matrix of the channel between the *r*th relay and the *p*th receiver
s_p	Transmitted signal by the *p*th source
v_r	Zero-mean additive white Gaussian noise (AWGN) at the *r*th relay node
p_p	Transmit power of the *p*th source
x_r	Received signal at the *r*th relay
$\mathbf{W}$	Beamforming matrix
n_k	Gaussian noise at the *k*th receiver
$p_{x,r}^{max}$	Maximum transmit power at the *r*th relay
p_x^{max}	Maximum total transmit power at relays
$\mathbf{R}_x$	Correlation matrix of the signal received at the relay
$\mathbf{R}_f^P$	Correlation matrix of the first hop channel gains
$\mathbf{R}_g^P$	Correlation matrix of the second hop channel gains
p_s^k	*k*th desired signal power
p_i^k	Interference power at kth destination
p_n^k	Noise power at kth destination
$\boldsymbol{h}_k$	Total path gain between the *k*th source and its corresponding destination
σ_f^2	Variance of the first hop channel coefficient ($\boldsymbol{f}$)
σ_g^2	Variance of the second hop channel coefficient ($\boldsymbol{g}$)
σ_n^2	Noise variance
$\bar{f}_{rp}$	Mean value of f_{rp}
ef_{rp}	Unknown part of f_{rp}
σ_{ef}^2	Variance of ef_{rp}
$\bar{g}_{rq}$	Mean value of g_{rq}
eg_{rq}	Unknown part of g_{rq}
σ_{eg}^2	Variance of eg_{rp}

III. SUM-RATE MAXIMIZATION CONSIDERING RELAY POWERS

In general, there are two ways to control the power of relays:

1- Case 1: Total sum of powers at all relays constraint.

2- Case 2: A set of individual power constraints at the relays

The first one is often considered in the cellular system literature while the second one is often considered in an ad-hoc network to extend the battery-powered relays' lifetime [23]. Assuming $p_{x,r}^{max}$ as the maximum transmit power at the *r*th relay and p_{x}^{max} as the maximum total transmit power at relays and considering power control, the first set of constraints are [23]:

$$\sum_{r=1}^{R} p_{x,r}^{max} \leq p_{x}^{max} \tag{5}$$

and the second constraint (Individual Relays' Powers Constraints) is:

$$p_{x,r} \leq p_{x,r}^{max} \quad , r = 1, 2, .., R \tag{6}$$

First, we study the first case in the subsection A and then investigate the second case in the subsection B.

A. Case 1 (Total Relay Power Constraint)

In the case 1, our objective is to achieve the weight of optimum beamforming $\{w_r^*\}_{r=1}^{R}$, in such a way that sum-rate become maximized while the total transmitted (consumed) power in relays is lower than the certain predefined threshold. Thus, the optimization problem can be written as:

$$\begin{aligned} \max \quad & \text{Sum} - \text{rate} \\ \text{subject to} \quad & \sum_{r=1}^{R} p_{x,r}^{max} \leq p_{x}^{max} \end{aligned} \tag{7}$$

where p_{x}^{max} is the predefined threshold as maximum allowable total transmitted power and $p_T = \sum_{r=1}^{R} p_{x,r}^{max}$ is the total transmitted power in all relay nodes. From (3), total transmitted power in relays can be given as (8).

$$\begin{aligned} p_T &= E\{\boldsymbol{t}^H \boldsymbol{t}\} \\ &= E\{\boldsymbol{x}^H \mathbf{W}\mathbf{W}^{\mathrm{H}} \boldsymbol{x}\} \\ &= tr\{\mathbf{W}^{\mathrm{H}} E\{\boldsymbol{x}\boldsymbol{x}^H\}\mathbf{W}\} \end{aligned} \tag{8}$$

where $tr(.)$ represents the trace of a matrix. Now, the total transmitted power can be rewritten of the form (9):

$$p_T = tr\{\mathbf{W}^{\mathrm{H}}\mathbf{R}_x\mathbf{W}\} = \sum_{r=1}^{R} |w_r|^2 [\mathbf{R}_x]_{r,r} = \boldsymbol{w}^{\mathrm{H}}\mathbf{D}\,\boldsymbol{w} \tag{9}$$

where $\mathbf{R}_x = E\{\boldsymbol{x}\boldsymbol{x}^H\}$ is the correlation matrix of the signal received at the relay, $\boldsymbol{w} = \text{diag}(\mathbf{W})$ and $\boldsymbol{D} = \text{diag}([\mathbf{R}_x]_{1,1}, [\mathbf{R}_x]_{2,2}, \ldots, [\mathbf{R}_x]_{R,R})$. Here, the vector $\text{diag}(\mathbf{A})$ contains the diagonal entries of the square matrix $\mathbf{A}$ and diagonal matrix $\text{diag}(\mathbf{a})$ is produced by the elements of the vector $\mathbf{a}$.

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Using (2) and respected assumptions 1-4, $\mathbf{R}_x$ can be reformulated as

$$\begin{aligned}\mathbf{R}_x &= \sum_{p,q=1}^{d} E\{\boldsymbol{f}_p\boldsymbol{f}_q^H\}\, E\{s_p s_q^*\} + \sigma_v^2\, \mathbf{I} \\ &= \sum_{p=1}^{d} P_p E\{\boldsymbol{f}_p\boldsymbol{f}_q^H\} + \sigma_v^2\, \mathbf{I} \\ &= \sum_{p=1}^{d} P_p \mathbf{R}_f^P + \sigma_v^2\, \mathbf{I}\end{aligned} \tag{10}$$

where $\mathbf{I}$ is the identity matrix, $\mathrm{s_p}$ and $\mathrm{s_q}$ are respectively the symbols transmitted by the pth and qth sources, and $\boldsymbol{R}_f^P$ is as (11):

$$\mathbf{R}_f^P = E\{\boldsymbol{f}_p\boldsymbol{f}_q^H\} \tag{11}$$

It is clear from (9) that the total transmit power of relays depends on the noise powers in relays and variance of the coefficients of the source-relay channel [22].

According to the variables that we described in the previous equations, sum-rate for the system shown in Figure 1 can be obtained from (12) [24]:

$$R_{\text{sum}} = \frac{1}{2}\sum_{k=1}^{d} \log_2 \det(1 + \mathbf{F}_k^{-1}\mathbf{T}_{kk}\mathbf{T}_{kk}^H) \tag{12}$$

where $\mathbf{T}_{kq} = \sum_{r=1}^{R} g_{kr} w_r f_{rq}$,$\bar{g}_{kr} = g_{kr}\, w_r$ and $\mathbf{F}_k = \sum_{q\in d\,,q\neq k} \mathbf{T}_{kq}\ \mathbf{T}_{kq}^H + \sum_{r=1}^{R} \bar{g}_{kr}\ \ \bar{g}_{kr}^H + \sigma_n^2$.

1/2 coefficient is appeared in (12) because two-phase scenario is considered. Authors of [23] and [24] showed that optimization of sum-rate is more complicated. According to theorem 1 expressed in [24], by introducing a lower bound as (13), the problem would be optimized more easily compared to the first problem, but the solution is suboptimal. Maximizing the lower bound compared to the previous research works [22-25] offers higher sum-rate which is the main idea of this investigation.

$$\log_2(1 + \text{TSTLR}) \le 2\, R_{\text{sum}} \tag{13}$$

In our investigation, destinations have single-antenna, so the condition of theorem 1 in [24] is satisfied.

TSTLR is defined as the total power of desired signals to total leakage power ratio, i.e.,

$$\text{TSTLR} = \frac{\sum_{k=1}^{d} p_s^k}{\sum_{k=1}^{d} (p_n^k + p_i^k)} \tag{14}$$

The desired signal power p_s^k, interference power p_i^k, and noise power p_n^k, can be achieved in terms of $\{w_r^*\}_{r=1}^{R}$. Using (4), for the noise power at the kth receiver, we have [22]:

$$\begin{aligned}p_n^k &= E\{\boldsymbol{v}^H\, \mathbf{W}\, \boldsymbol{g}_k^*\, \boldsymbol{g}_k^T\, \mathbf{W}^{\mathrm{H}}\, \boldsymbol{v}\} + \sigma_n^2 \\ &= tr\{\mathbf{W}^{\mathrm{H}}\, E\{\boldsymbol{v}\boldsymbol{v}^H\}\quad \mathbf{W}\ \ E\{\boldsymbol{g}_k^*\, \boldsymbol{g}_k^T\}\ \} + \sigma_n^2\end{aligned} \tag{15}$$

$$= \sigma_v^2 \, tr\{ \mathbf{W}^{\mathrm{H}} \mathbf{R}_g^k \, \mathbf{W} \quad \} + \sigma_n^2$$

where $\mathbf{R}_g^k = E\{\boldsymbol{g}_k \boldsymbol{g}_k^H\}$.

The *k*th receiver noise power can be rewritten as:

$$p_n^k = \sigma_v^2 \sum_{r=1}^{R} |w_r|^2 \left[\mathbf{R}_g^k\right]_{r,r} + \sigma_n^2$$
$$= \boldsymbol{w}^H \mathbf{D}_k \boldsymbol{w} + \sigma_n^2 \qquad (16)$$

where $\mathbf{D}_k = \sigma_v^2 \, diag \, (\left[\mathbf{R}_g^k\right]_{1,1}, \left[\mathbf{R}_g^k\right]_{2,2}, \dots, \left[\mathbf{R}_g^k\right]_{R,R})$.

The *k*th desired signal power can be expressed as:

$$p_s^k = E\{\boldsymbol{g}_k^T \, \mathbf{W}^{\mathrm{H}} \boldsymbol{f}_k \, \boldsymbol{f}_k^H \, \mathbf{W} \, \boldsymbol{g}_k^* \; \} E\{|s_k|^2\}$$
$$= p_k E\{\boldsymbol{w}^{\mathrm{H}} \, \mathrm{diag}(\boldsymbol{g}_k) \, \boldsymbol{f}_k \, \boldsymbol{f}_k^H \, \mathrm{diag}(\boldsymbol{g}_k^*) \mathbf{W} \; \} \qquad (17)$$
$$= p_k E\{\boldsymbol{w}^{\mathrm{H}} \, (\boldsymbol{g}_k \odot \boldsymbol{f}_k) \, (\boldsymbol{f}_k^H \odot \boldsymbol{g}_k^H) \boldsymbol{w} \; \}$$
$$= p_k \, \boldsymbol{w}^{\mathrm{H}} \, E\{ \; \boldsymbol{h}_k \boldsymbol{h}_k^H \; \} \boldsymbol{w}$$
$$= \boldsymbol{w}^{\mathrm{H}} \, \mathbf{R}_h^k \, \boldsymbol{w}$$

where $\odot$ is element-wise Schur-Hadamard multiplication of two matrices.

$$\boldsymbol{h}_k = (\boldsymbol{g}_k \odot \boldsymbol{f}_k) = [g_{1k} \, f_{1k} \, , g_{2k} f_{2k} \; , \dots , g_{Rk} \, f_{Rk}]^T$$
$$\mathbf{R}_h^k = p_k E\{ \; \boldsymbol{h}_k \boldsymbol{h}_k^H \; \} \qquad (18)$$

As shown in (18), $\boldsymbol{h}_k$ contains the total path gains between the *k*th source and its corresponding destination considering different relays. Also, using (4) and denoting $A_k = \{1,2,\dots,d\} - \{k\}$, the interference power is given by [22]:

$$p_i^k = E\{\boldsymbol{g}_k^T \, \mathbf{W}^{\mathrm{H}} \left(\sum_{p,q \in A_k} \boldsymbol{f}_p \boldsymbol{f}_q^H \; s_p s_q^* \right) \mathbf{W} \, \boldsymbol{g}_k^* \}$$
$$= E\{ \, \boldsymbol{w}^{\mathrm{H}} \, \mathrm{diag} \, (\boldsymbol{g}_k) \left(\sum_{p \in A_k} p_p \boldsymbol{f}_p \boldsymbol{f}_p^H \right) \mathrm{diag}(\boldsymbol{g}_k^*) \, \boldsymbol{w} \, \}$$
$$= E\{ \, \boldsymbol{w}^{\mathrm{H}} \left(\sum_{p \in A_k} p_p (\boldsymbol{g}_k \otimes \boldsymbol{f}_p)(\boldsymbol{g}_k^H \otimes \boldsymbol{f}_p^H) \right) \boldsymbol{w} \, \}$$
$$= \boldsymbol{w}^{\mathrm{H}} \, \mathbf{Q}_k \, \boldsymbol{w} \qquad (19)$$

where $\boldsymbol{h}_k^p$ and $\mathbf{Q}_k$ are defined as:

$$\boldsymbol{h}_k^p = (\boldsymbol{g}_k \otimes \boldsymbol{f}_p)$$
$$\mathbf{Q}_k = E\left\{ \sum_{p \in A_k} p_p \, \boldsymbol{h}_k^p \left(\boldsymbol{h}_k^p \right)^H \right\} \qquad (20)$$

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By using (16), (17) and (19), we can rewrite (14) as (21):

$$\text{TSTLR} = \frac{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}\boldsymbol{R}_h^k\right)\boldsymbol{w}}{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}\mathbf{Q}_k\right)\boldsymbol{w} + \boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}\mathbf{D}_k\right)\boldsymbol{w} + \boldsymbol{d}\,\sigma_n^2} = \frac{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}\boldsymbol{R}_h^k\right)\boldsymbol{w}}{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}(\mathbf{Q}_k+\mathbf{D}_{k)}\right)\boldsymbol{w} + d\sigma_n^2} \tag{21}$$

The direct optimization of the system sum-rate is complicated. Therefore, we approximate it by the TSTLR and maximize it. Hence, the optimization problem can be rewritten as:

$$\begin{aligned} &\max \quad \text{TSTLR} \\ &\text{subject to} \quad p_T \le p_x^{max} \end{aligned} \tag{22}$$

Using the previous equations, we can write:

$$\begin{aligned} &\max \quad \frac{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}\mathbf{R}_{\text{h}}^{\text{k}}\right)\boldsymbol{w}}{\boldsymbol{w}^{\text{H}}\left(\sum_{k=1}^{d}(\mathbf{Q}_k+\mathbf{D}_{k)}\right)\boldsymbol{w} + d\sigma_n^2} \\ &\text{subject to} \quad \boldsymbol{w}^{\text{H}}\,\mathbf{D}\,\boldsymbol{w} \le p_x^{max} \end{aligned} \tag{23}$$

There is no efficient method to deal directly with (23) because the objective function is a fractional relationship. This makes (23) to be a hard problem because it is a non-convex problem with high complexity solution.

We exploit a SDP relaxation approach to find a relaxed version of (23). By defining $\mathbf{X} = \boldsymbol{w}\,\boldsymbol{w}^H$ [21, 22], the problem (23) can be changed to (24):

$$\begin{aligned} &\max \quad \text{tr}\,(\mathbf{ZX}) \\ &\text{subject to} \quad tr\,(\mathbf{DX}) \le p_x^{max} \\ &\text{rank}\,(\mathbf{X}) = 1\,, \quad \mathbf{X} \ge 0 \end{aligned} \tag{24}$$

where $\mathbf{X} \ge 0$ means that $\mathbf{X}$ is a positive semi definite (PSD) matrix, and rank (.) denotes the rank of a matrix.

To fix this problem, there are three ways as:

1. Minimizing the denominator ($\mathbf{Z} = \sum_{k=1}^{d}(\mathbf{Q}_k + \mathbf{D}_k)$)
2. Maximizing the numerator ($\mathbf{Z} = \sum_{k=1}^{d}\mathbf{R}_{\text{h}}^{\text{k}}$)
3. Maximizing the difference between the numerator and the denominator ($\mathbf{Z} = \sum_{k=1}^{d}\{\mathbf{R}_{\text{h}}^{\text{k}} - (\mathbf{Q}_k + \mathbf{D}_k)\}$).

According to simulation results [25], maximizing TSTLR by maximizing the difference between the numerator and the denominator, ($\mathbf{Z} = \sum_{k=1}^{d}\{\mathbf{R}_{\text{h}}^{\text{k}} - (\mathbf{Q}_k + \mathbf{D}_k)\}$), introduces higher sum-rate with respect to two other ones in all SNRs. Hence, the difference between the numerator and the denominator is considered as objective function.

As the next problem, the rank constraint in (24) is non-convex. By removing this non-convex constraint using SDR we have:

$$\begin{aligned} &\max \quad \mathrm{tr}\,(\mathbf{ZX}) \\ &\text{subject to} \quad tr\,(\mathbf{DX}) \leq p_x^{max} \\ &\mathbf{X} \geq 0 \end{aligned} \tag{25}$$

Indeed, the problem (25) is convex and can be solved efficiently using convex optimization (CVX) MATLAB toolbox [30].

The rank-one property for the solution of problem (25), the matrix $\mathbf{X}_{\mathrm{opt}}$, is not guaranteed. It means that the solution for problem (25) only provides an upper bound for problem (24). Proof of this is available in [22]. As shown in [31], the rank-one solution for problem (25) can be found as long as $d \leq 3$. For the case $d > 3$, to obtain a suboptimal rank-one solution, randomization techniques should be used. Here, $\mathrm{X}_{\mathrm{opt}}$ is used to make suboptimal weight vectors $\boldsymbol{w}$ from which the best solution will be selected [32, 33, 34].

In order to solve (23), assume $\boldsymbol{w}$ as:

$$\boldsymbol{w} = \sqrt{p}\,\mathbf{D}^{-\frac{1}{2}}\,\widetilde{\boldsymbol{w}} \tag{26}$$

where $\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\boldsymbol{w}} = \mathbf{1}$.The optimization problem (23) can be represented as:

$$\begin{aligned} &\max \quad \frac{p\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\mathbf{R}}\,\widetilde{\boldsymbol{w}}}{p\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\mathbf{H}}\,\widetilde{\boldsymbol{w}} + d\,\sigma_n^2} \\ &\text{subject to} \quad \|\widetilde{\boldsymbol{w}}\|^2 = 1 \quad , p \leq p_x^{max} \end{aligned} \tag{27}$$

where $\widetilde{\mathbf{R}} = \mathbf{D}^{-\frac{1}{2}}\left(\sum_{k=1}^{d}\mathbf{R}_h^k\right)\mathbf{D}^{-\frac{1}{2}}$ and $\widetilde{\mathbf{H}} = \mathbf{D}^{-\frac{1}{2}}\left(\sum_{k=1}^{d}(\mathbf{Q}_k + \mathbf{D}_k)\right)\mathbf{D}^{-\frac{1}{2}}$.

This objective function will be maximized for $p = p_x^{max}$ because the objective function in (27) is monotonically increasing in p. So, we can simplify the problem (27) as:

$$\begin{aligned} &\max \quad \frac{p_x^{max}\,\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\mathbf{R}}\,\widetilde{\boldsymbol{w}}}{p_x^{max}\,\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\mathbf{H}}\,\widetilde{\boldsymbol{w}} + d\,\sigma_n^2} \\ &\text{subject to} \quad \|\widetilde{\boldsymbol{w}}\|^2 = 1 \end{aligned} \tag{28}$$

Or we can equivalently write:

$$\begin{aligned} &\max \quad \frac{p_x^{max}\,\widetilde{\boldsymbol{w}}^{\mathrm{H}}\widetilde{\mathbf{R}}\,\widetilde{\boldsymbol{w}}}{p_x^{max}\widetilde{\boldsymbol{w}}^{\mathrm{H}}\left(\widetilde{\mathbf{H}} + d\,\mathbf{I}\,\sigma_n^2\right)\widetilde{\boldsymbol{w}}} \\ &\text{subject to} \quad \|\widetilde{\boldsymbol{w}}\|^2 = 1 \end{aligned} \tag{29}$$

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It is well known [35] that the objective function in (29) is globally maximized when $\tilde{\boldsymbol{w}}$ is chosen as the principal generalized eigenvector of $\left(\tilde{\mathbf{R}}\,,\, d\,\mathbf{I}\,\sigma_n^2 + \, p_x^{max}\,\tilde{\mathbf{H}}\right)$, or, equivalently, as the principal eigenvector of the matrix $\left(\,d\,\mathbf{I}\,\sigma_n^2 + \, p_x^{max}\,\tilde{\mathbf{H}}\right)^{-1}\tilde{\mathbf{R}}$. In order to satisfy the unit-norm constraint in (29), the normalized version of problem (29) will be considered which it has the following solution

$$\tilde{\boldsymbol{w}} = p.e.v\left\{\left(\,d\,\mathbf{I}\,\sigma_n^2 + \, p_x^{max}\,\tilde{\mathbf{H}}\right)^{-1}\tilde{\mathbf{R}}\right\} \tag{30}$$

where $p.e.v\{\cdot\}$ is the normalized principal eigenvector of a matrix. Finally, we have

$$\boldsymbol{w} = \sqrt{p_x^{max}}\,\mathbf{D}^{-\frac{1}{2}}\,p.e.v\left\{\left(\,d\,\mathbf{I}\,\sigma_n^2 + \, p_x^{max}\,\tilde{\mathbf{H}}\right)^{-1}\tilde{\mathbf{R}}\right\} \tag{31}$$

B. Case 2 (Individual Relays' Powers Constraints)

In this subsection, we consider the case that relay transmit power for each relay node is constrained. When the battery lifetime for relay nodes is limited, this case is more appropriate. So, our goal is finding the solution for problem (32).

$$\begin{aligned} &\max \quad \text{Sum} - \text{rate} \\ &p_{x,r} \le p_{x,r}^{max} \quad , r = 1, 2, .., R \end{aligned} \tag{32}$$

By comparing optimization problems (22) and (32), it is clear that the total sum of relay powers constraint is replaced by R individual relay power constraints. Again using (19), we can write:

$$\begin{aligned} &\max \quad \text{TSTLR} \\ &p_{x,r} \le p_{x,r}^{max} \quad , r = 1, 2, .., R \end{aligned} \tag{33}$$

Using (21), we can rewrite (33) as:

$$\begin{aligned} &\max \quad \frac{\boldsymbol{w}^{\mathrm{H}}\left(\sum_{k=1}^{d}\mathbf{R}_{\mathrm{h}}^{\mathrm{k}}\right)\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\left(\sum_{k=1}^{d}(\mathbf{Q}_k + \mathbf{D}_k)\right)\boldsymbol{w} + d\sigma_n^2} \\ &\text{subject to} \quad \mathbf{D}_{\mathrm{ii}}\,|w_i|^2 \le p_{x,i}^{max} \quad i = 1,2,\dots,R \end{aligned} \tag{34}$$

where $\mathbf{D}_{\mathrm{ii}}$ is the *i*th diagonal entry of the matrix and $p_{x,i}^{max}$ is the maximum allowable power of the *i*th relay.

Denoting $\mathbf{X} = \boldsymbol{w}\,\boldsymbol{w}^H$, $\mathbf{H}_{\mathrm{t}} = \sum_{k=1}^{d}(\mathbf{Q}_k + \mathbf{D}_k)$ and $\mathbf{R}_{\mathrm{t}} = \sum_{k=1}^{d}\mathbf{R}_{\mathrm{h}}^{\mathrm{k}}$,(34) can be reformulated as

$$\begin{aligned} &\max \quad \frac{\mathrm{tr}\,(\mathbf{R}_{\mathrm{t}}\,\mathbf{X})}{\mathrm{tr}\,(\mathbf{H}_{\mathrm{t}}\,\mathbf{X}) + d\sigma_n^2} \\ &\text{subject to} \quad \mathbf{D}_{\mathrm{ii}}\,|w_i|^2 \le p_{x,i}^{max} \quad i = 1,2,\dots,R \\ &\text{rank}\,(\mathbf{X}) = 1, \mathbf{X} \ge 0 \end{aligned} \tag{35}$$

We can use IP-based methods to solve the convex form of the non-convex problem (35). For example, the CVX [30] as an IP-based software package produces a feasibility certificate for feasible problems. In SDR, the solution is not rank-one in general but we can find a bound. To do it, we consider the problem (36)

$$\max \quad \frac{\boldsymbol{w}^{\mathrm{H}}\left(\sum_{k=1}^{d}\mathbf{R}_{\mathrm{h}}^{\mathrm{k}}\right)\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\left(\sum_{k=1}^{d}(\mathbf{Q}_k+\mathbf{D}_k)\right)\boldsymbol{w}+d\sigma_n^2}$$
$$\text{subject to} \quad \boldsymbol{w}^{\mathrm{H}}\mathbf{G}_{\mathrm{i}}\boldsymbol{w}\le 1 \quad i=1,2,\dots,R \tag{36}$$

where $\mathbf{G}_{\mathrm{i}}$ is a matrix which the elements located on the ith diagonal are $\frac{\mathbf{D}_{ii}}{p_{x,i}^{max}}$ and the others are zero. As before, considering the above-mentioned assumptions, (36) can be rewritten as:

$$\max \quad \frac{\operatorname{tr}(\mathbf{R}_{\mathrm{t}}\,\mathbf{X})}{\operatorname{tr}(\mathbf{H}_{\mathrm{t}}\,\mathbf{X})+d\sigma_n^2}$$
$$\text{subject to} \quad \operatorname{tr}(\mathbf{G}_{\mathrm{i}}\mathbf{X})\le 1 \quad i=1,2,\dots,R$$
$$\operatorname{rank}(\mathbf{X})=1\,,\ \ \mathbf{X}\ge 0 \tag{37}$$

Using the SDP relaxation, we can write:

$$\max \quad \frac{\operatorname{tr}(\mathbf{R}_{\mathrm{t}}\,\mathbf{X})}{\operatorname{tr}(\mathbf{H}_{\mathrm{t}}\,\mathbf{X})+d\sigma_n^2}$$
$$\text{subject to} \quad \operatorname{tr}(\mathbf{G}_{\mathrm{i}}\mathbf{X})\le 1 \quad i=1,2,\dots,R$$
$$\mathbf{X}\ge 0 \tag{38}$$

In order to make an optimal $\mathbf{X}^{*}\ge 0$, the SDP relaxation can be solved in polynomial time by using bisection. In addition

$$\operatorname{tr}(\mathbf{R}_{\mathrm{t}}\,\mathbf{X}^{*})=\mu^{*}(\operatorname{tr}(\mathbf{H}_{\mathrm{t}}\,\mathbf{X}^{*})+d\sigma_n^2) \tag{39}$$

It is obvious that μ^{*} is an upper bound for the problem (36). Here, the nonconvex quadratic problem can be changed to:

$$\max \quad \boldsymbol{w}^{\mathrm{H}}\,\mathbf{R}_{\mathrm{t}}\,\boldsymbol{w}-\mu^{*}\left(\boldsymbol{w}^{\mathrm{H}}\,\mathbf{H}_{\mathrm{t}}\,\boldsymbol{w}+d\sigma_n^2\right)$$
$$\text{subject to} \quad \boldsymbol{w}^{\mathrm{H}}\,\mathbf{G}_{\mathrm{i}}\,\boldsymbol{w}\le 1 \quad i=1,2,\dots,R \tag{40}$$

Now, we can write its SDP relaxation as:

$$\max \quad \operatorname{tr}(\mathbf{R}_{\mathrm{t}}\,\mathbf{X})-\mu^{*}(\operatorname{tr}(\mathbf{H}_{\mathrm{t}}\,\mathbf{X})+d\sigma_n^2)$$
$$\text{subject to} \quad \operatorname{tr}(\mathbf{G}_{\mathrm{i}}\mathbf{X})\le 1 \quad i=1,2,\dots,R$$
$$\mathbf{X}\ge 0 \tag{41}$$

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It follows that $\mathbf{X}^* \geq 0$ is a global optimal solution for (41). By considering a complex Gaussian distribution $N(0, \mathbf{X}^*)$ and according to [36], we can find an approximate solution for $\widehat{\boldsymbol{w}}$ in randomized polynomial time which satisfies:

$$\widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{R}_{\mathrm{t}} \widehat{\boldsymbol{w}} - \mu^* \widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{H}_{\mathrm{t}} \widehat{\boldsymbol{w}} \geq c \left(tr \left(\mathbf{R}_{\mathrm{t}} \mathbf{X}^* \right) - \mu^* \, tr(\mathbf{H}_{\mathrm{t}} \mathbf{X}^*) \right) \tag{42}$$

where $c = \mathrm{O}((\log R)^{-1})$ is a constant. According to (38), we obtain:

$$\widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{R}_{\mathrm{t}} \widehat{\boldsymbol{w}} - \mu^* \widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{H}_{\mathrm{t}} \widehat{\boldsymbol{w}} \geq c \mu^* \, d\sigma_n^2 \tag{43}$$

Indicates that

$$\widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{R}_{\mathrm{t}} \widehat{\boldsymbol{w}} - \mu^* \widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{H}_{\mathrm{t}} \widehat{\boldsymbol{w}} \geq c \mu^* \, d\sigma_n^2 + (1-c) \mu^* \widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{H}_{\mathrm{t}} \widehat{\boldsymbol{w}} \geq c \mu^* \, d\sigma_n^2 \tag{44}$$

where the last step follows from the positive semidefiniteness of $\mathbf{H}_{\mathrm{t}}$. Rearranging the terms, we obtain:

$$\frac{\widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{R}_{\mathrm{t}} \widehat{\boldsymbol{w}}}{\widehat{\boldsymbol{w}}^{\boldsymbol{H}} \mathbf{H}_{\mathrm{t}} \widehat{\boldsymbol{w}} + d\sigma_n^2} \geq c \mu^* \tag{45}$$

It indicates that $\widehat{\boldsymbol{w}}$ is an optimal solution of (36) in order c [37]. It means that by using SDP relaxation technique, an $c = \mathrm{O}((\log R)^{-1})$ approximation would be found for nonconvex fractional quadratic optimization problem (36).

As mentioned above, the optimization problem for sum-rate maximization considering individual relay power constraints can be feasible by using an iterative procedure where a convex feasibility problem would be solved in each step. We are looking for a solution with reasonable computational complexity instead of solving (35) or (36) directly. More details are available in appendix 1. One simple solution is to turn (34) into a low-complex problem. Hence, by ignoring $d\sigma_n^2$ in (34), the following problem should be solved:

$$\begin{aligned} &\max \quad \frac{\boldsymbol{w}^{\mathrm{H}} \left(\sum_{k=1}^{d} \mathbf{R}_{\mathrm{h}}^{\mathrm{k}} \right) \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \left(\sum_{k=1}^{d} (\mathbf{Q}_k + \mathbf{D}_k) \right) \boldsymbol{w}} \\ &\text{subject to} \quad \mathbf{D}_{\mathrm{ii}} \, |w_i|^2 \leq p_{x,i}^{max} \quad i = 1,2,\dots,R \end{aligned} \tag{46}$$

Indeed, the objective function of problem (46) is an upper bound for the objective function of problem (34) which is globally maximized for

$$\boldsymbol{w} = \eta \boldsymbol{\nu} \tag{47}$$

where $\boldsymbol{\nu}$ denotes the normalized principal eigenvector of the matrix $\mathbf{H}_{\mathrm{t}}^{-1} \mathbf{R}_{\mathrm{t}}$ and η is any scalar parameter [38] which can be chosen as

$$\eta = \frac{1}{\left(\sqrt{\mathbf{D}_{k,k}} \, |\nu_k| / \sqrt{p_{x,k}^{max}} \right)} \tag{48}$$

where ν_k denotes the *k*th entry of $\boldsymbol{\nu}$ and

$$k = \arg\max \frac{\mathbf{D}_{i,i}\ \boldsymbol{\nu}_i^2}{p_{x,i}^{max}} \quad 1 \le i \le R \tag{49}$$

Therefore, the global maximum for (47) is the good solution for (46).

IV. SIMULATION RESULTS

In this section, the impact of various factors on the performance of the proposed schemes is shown for both total power constraint and individual relay power constraints. Also, assume that after solving the optimization problem and finding the beamforming weights as well as proper relay powers, communication between source-destination pairs through respected relay nodes can be done. Beamforming weights and relay powers are fixed up to the end of connection. It means, there is no need to further updating the relay powers and do the optimization problem during service time. In actual scenario, this case is valid if the source-destination and relay nodes are stationary or quasi-static.

In all simulations, it is assumed that maximum total power consumption at relays is equal to unit ($p_x^{max} = 1$). Also, in individual power constrained case, we set the maximum allowable power consumption equal to $\frac{1}{R}$ (i.e., $p_{x,r}^{max} = \frac{1}{R}\ \ r = 1, 2, .., R$).

The channel coefficients $\boldsymbol{f}$ and $\boldsymbol{g}$ are generated as identically independent distribution (i.i.d) complex Gaussian random variables with variances σ_f^2 and σ_g^2, respectively. It is also assumed that all transmitters have equal unit powers. All simulation results are averaged over 500 independent channel realizations. All MATLAB simulation codes are run on a PC with RAM=4GB, Processor: Intel (R) Core ™ i5-2400 CPU @ 3.1GHz, System Type: 64bit.

A. Comparison of Sum-Rate Maximization Methods

In the first experiment, five methods to maximize sum-rate are compared in the case of the two pairs of source-destination (*d*=2) considering 20 relays (*R*=20) and $\sigma_f^2 = \sigma_g^2 = 10\ dB$. As depicted in Figure 2, the proposed total power constrained method offers the highest sum-rate with respect to the other methods in all SNRs.

It should be noted that for low SNRs (high noise variances), the performance of total power constrained methods (proposed and [24], [25]) is the same and also their values are greater than the performance of two others. This gain is due to having more freedom in power allocation in the sum-power constrained case as relays may transmit at a higher value than the maximum transmit power at a relay in the individual power constrained case. This means that extra constraints added by the individual power constraints at the relays have little impact on the end-to-end sum-rate performance of

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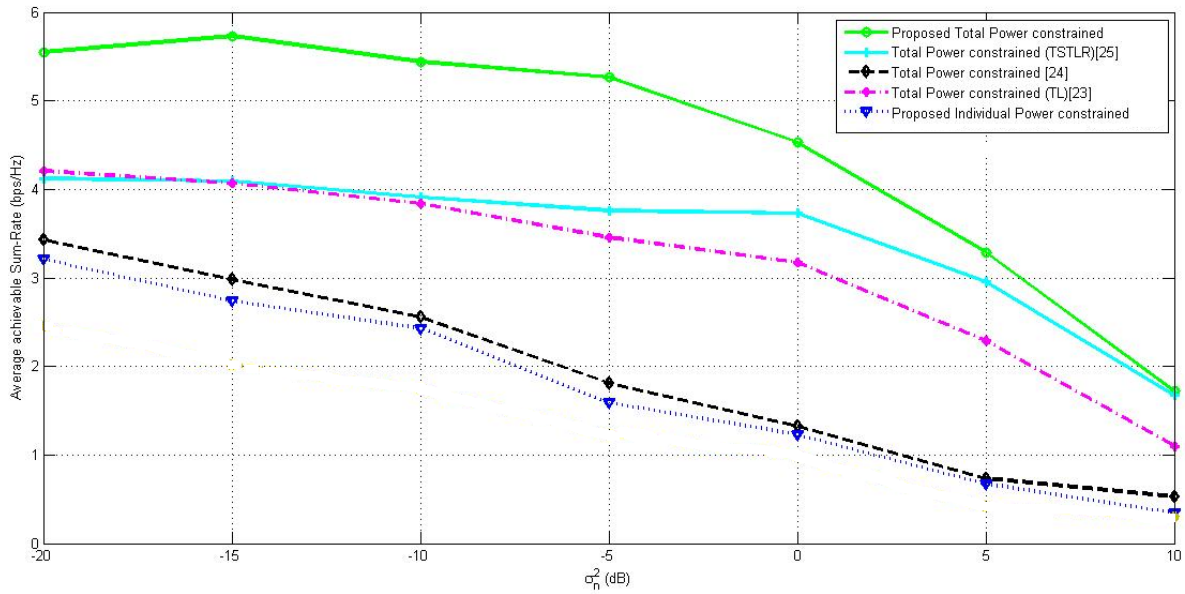

Fig 2. Average achieved sum-rate versus noise variance for different relay power constrained sum-rate maximization methods.

the proposed algorithms (the relay sum power constraint is replaced by R individual power constraints). In contrast, for high SNRs (low noise variances), the performance of the proposed total power constrained method is effectively greater than that for [23]-[25].

In order to have a fair comparison, the energy efficiency versus noise variance for our proposed algorithms and schemes in [22], [23], [24] and [25] are plotted in Fig 3. As shown in this figure, our proposed algorithms are more energy-efficient than the others. For example, when σ_n^2 is 0 dB, the energy efficiency gap between the proposed algorithms and schemes proposed in [22] , [23] ,[24] and [25] is respectively about 1.3 (b/s/Hz/W), 3.2 (b/s/Hz/W)), 1.1 (b/s/Hz/W) and 1 (b/s/Hz/W), which shows about 30%, 70%, 70%, and 22% improvements, respectively.

In order to have a view on running time, computational complexity of the proposed schemes are compared to two algorithms proposed in [23] and [25]. Table II shows this comparison when two pairs of source-destination (*d*=2) communicate with each other through 20 relays (*R*=20) in the case of $\sigma_f^2 = \sigma_g^2 = 10\ dB$ and $\sigma_n^2 = 0\ dB$.

As it is shown in Table II, the complexity of the proposed algorithms (total and individual relay power constraints schemes) are lower than TSTLR and TL algorithms. Considering the simulation results of Fig. 2 and Table II, TL algorithm [23] offers lower performance and needs higher running time respect to the other algorithms. Therefore, in the next experiments this method is not examined.

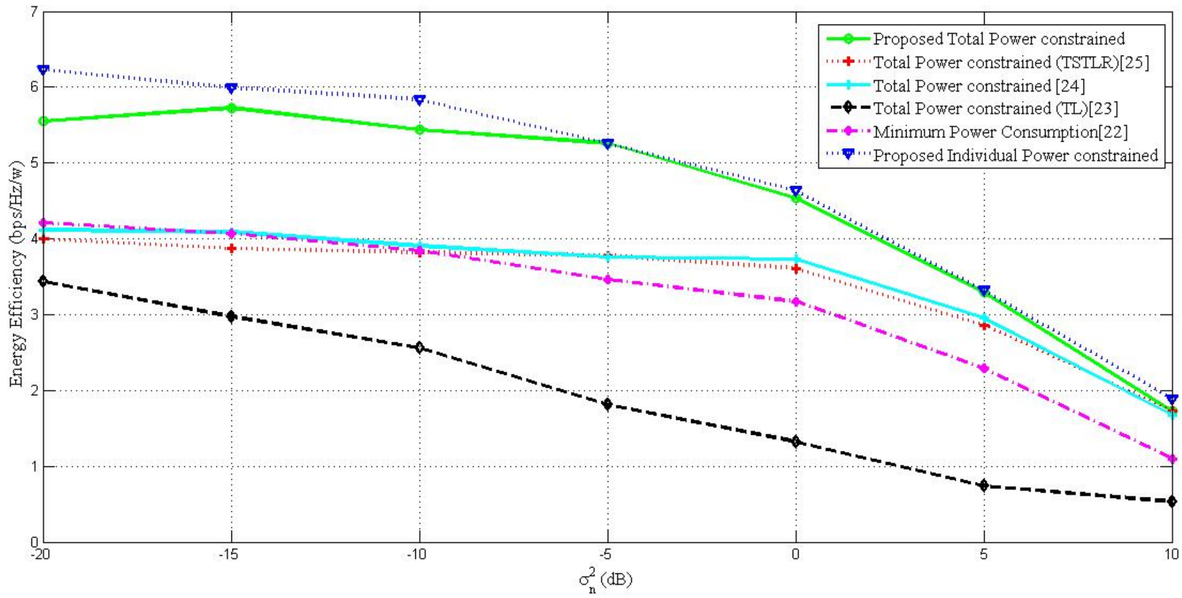

Fig 3. Energy efficiency versus noise variance for different schemes

Table II. Relative running time of different sum-rate maximization schemes

Optimization algorithm	**Relative running time** (Compared to reference one)
The proposed scheme 1: Total relay power constraint	1 (reference)
The proposed scheme 2: Individual relay power constraints	3.34
TL scheme [23]: Total relay transmit power constraint	705.34
Scheme [24]: Total relay transmit power constraint	712.59
TSTLR scheme [25]: Total relay transmit power constraint	693.45

B. The Effect of the Number of Relays in the Performance of the Proposed Algorithms

Figure 4 illustrates sum-rate in terms of noise power, σ_n^2, for different numbers of relays, when *d*=2 and $\sigma_f^2 = \sigma_g^2 = 10\ dB$. It is obvious that if the number of relays increases, the achievable sum-rate will be increased.

In addition, higher number of relays provides additional higher diversity. It should be mentioned that higher number of relays introduces more computational complexity. It is interesting to note that although we obtain higher sum-rate for a specific noise variance using higher number of relays, the sum-rate improvement for the same difference number of relays are not the same. For example, in $\sigma_n^2 = -10\ dB$, the sum-rate improvement for higher number of relays is summarized in Table III.

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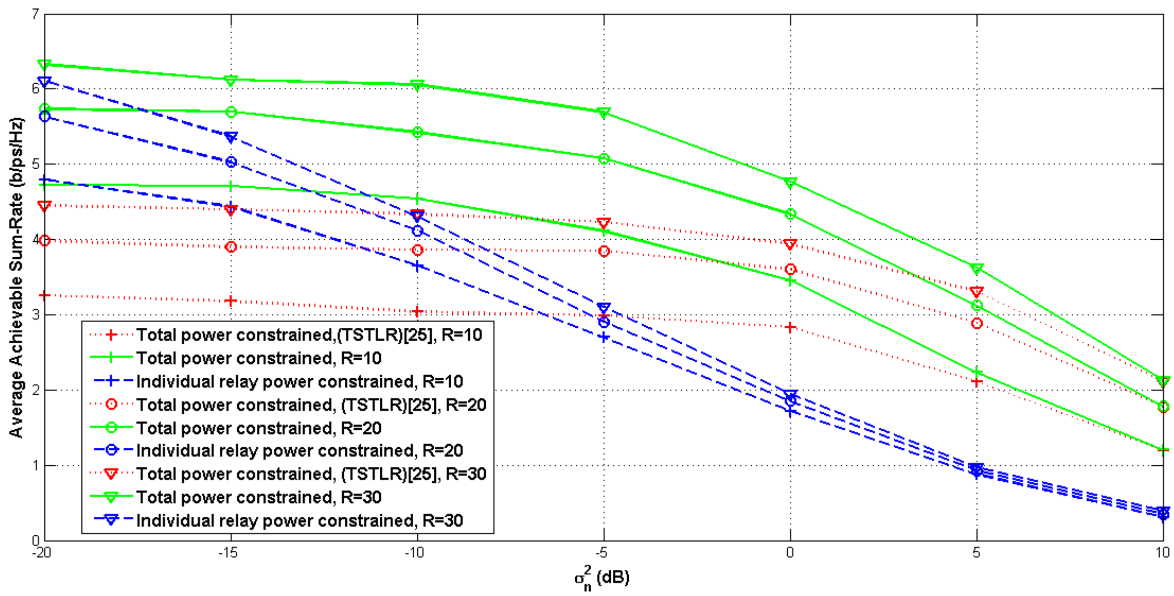

Fig 4. Average achieved sum-rate versus noise variance for different number of relays.

Table III. Sum-rate improvement for different number of relays

Method	**Sum-rate improvement (b/s/Hz)**	
	R=20 compared to R=10	R=30 compared to R=20
Total power constrained	0.8869	0.6314
Total power constrained, TSTLR [25]	0.8306	0.4701
Individual relay power constrained	0.4645	0.1949

C. *The Effect of Channel Quality*

In two following experiments, the effect of the uplink and downlink channel qualities is examined. By increasing channel variance or equivalently the quality of channel, average received signal power will be increased. It is obvious that when the channel variance (gain) of channel is increased, the average received signal power will be increased too.

In Figure 5, average achievable sum-rate is plotted versus $\sigma_n^2$where *R*=20, *d*=2 and $\sigma_g^2 = 10\ dB$ for different values of σ_f^2 . As shown in this Figure, by increasing σ_f^2 sum-rate will be increased. In the case of *R*=20, *d*=2 and $\sigma_f^2 = 10\ dB$ for different values of σ_g^2, the average achieved sum-rate

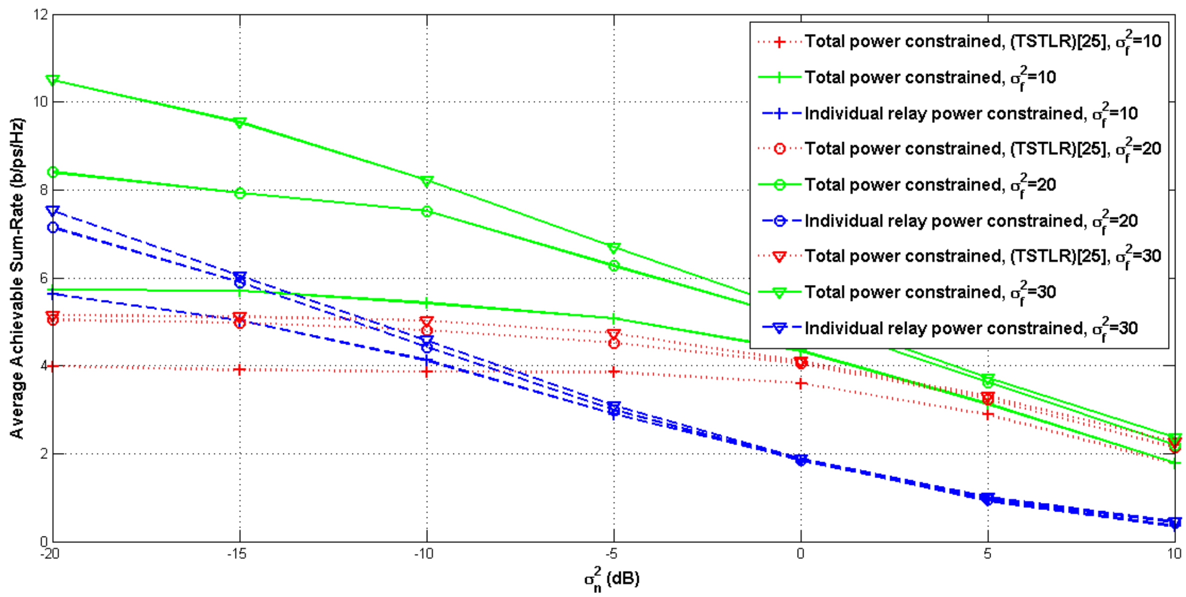

Fig 5. Average achieved sum-rate versus noise variance for different values of σ_f^2.

versus σ_n^2 is depicted in Figure 6. As shown, increasing the quality of downlink or uplink causes increasing the achievable sum-rate.

D. The Effect of Imperfect CSI

In the next experiment, the effect of imperfect CSI in our proposed algorithm is investigated. We assume a network consisting 20 relays (R=20). Also, It is supposed that the channel coefficients f_{rp} and $g_{r'q}$ are independent from each other for any p, q, r and r'. Also, we assume that the channel coefficient f_{rp} can be written as $f_{rp} = \bar{f}_{rp} + ef_{rp}$ where $\bar{f}_{rp}$ is the known mean value of f_{rp} and ef_{rp} is a zero-mean random variable with variance σ_{ef}^2 [22]. It is assumed that ef_{rp} and $ef_{r'p}$ are independent for any $r \neq r'$. We generate $\bar{f}_{rp} = \sqrt{1 - \sigma_{ef}^2}\, e^{j\theta}$, where θ is uniformly distributed in the interval $[0,2\pi]$. Since $E\left\{\left|f_{rp}\right|^2\right\} = 1$, by increasing σ_{ef}^2 , the mean value, $\bar{f}_{rp}$ will be decreased. It means that the uncertainty in the channel coefficient f_{rp} is increased. Similarly, we model the channel coefficient g_{rq} as $g_{rq} = \bar{g}_{rq} + eg_{rq}$ where $\bar{g}_{rq}$ is the known mean value of $g_{rq}$and eg_{rq} is a zero-mean random variable with variance σ_{eg}^2. It is assumed that eg_{rq} and $eg_{r'q}$ are independent for any $r \neq r'$. We consider $\bar{g}_{rq} = \sqrt{1 - \sigma_{eg}^2}\, e^{j\alpha}$, where α is a uniformly distributed random variable belong to the interval $[0,2\pi]$. Here, σ_{eg}^2 shows the level of uncertainty in the channel coefficient, g_{rq}. Based on this channel modeling, we can write the (r, r') entry matrices as:

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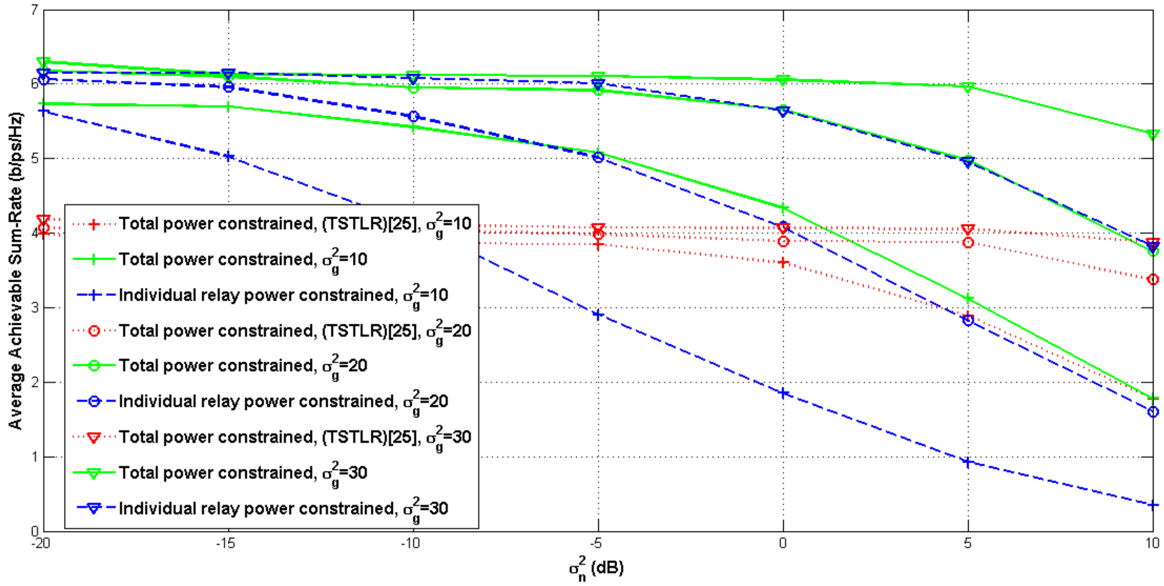

Fig 6. Average achieved sum-rate versus noise variance for different values of σ_g^2.

$$\mathbf{R}_f^p = \bar{\boldsymbol{f}}\,\bar{\boldsymbol{f}}^H + \sigma_{ef}^2\mathbf{I} \tag{49}$$

$$\mathbf{R}_g^p = \bar{\boldsymbol{g}}\,\bar{\boldsymbol{g}}^H + \sigma_{eg}^2\mathbf{I} \tag{50}$$

$$\mathbf{R}_h^k(r,r') = p_k\mathbf{R}_f^k(r,r').\mathbf{R}_g^k(r,r') \tag{51}$$

$$\mathbf{Q}_k(r,r') = \sum_{1,k\neq p}^{d} p_p\mathbf{R}_f^p(r,r').\mathbf{R}_g^k(r,r') \tag{52}$$

In this experiment, we choose the source power equal to 0dB. The average sum-rate versus σ_n^2 is plotted in the case of *R*=20 and *d*=2 for different values of σ_{ef}^2 and σ_{eg}^2 in Figure 7. This figure shows that increasing the uncertainty of the channels, σ_{ef}^2 and σ_{eg}^2, is the reason for decreasing the achievable sum-rate.

V. CONCLUSION

In this investigation, the problem of distributed beamforming in a network consisting of two pairs of transmitter-receiver and *R* relay nodes was solved by maximizing the achievable sum-rate subject to constraints on total and individual relay powers. Herein, we found a closed-form solution for the optimization problem with the total power constraint and showed that the problem with individual relay power constraints is a quadratic programming optimization problem which does not have a closed-form solution. We developed and presented a low-complex simplified suboptimal technique for solving the optimization problem including individual relay power constraints.

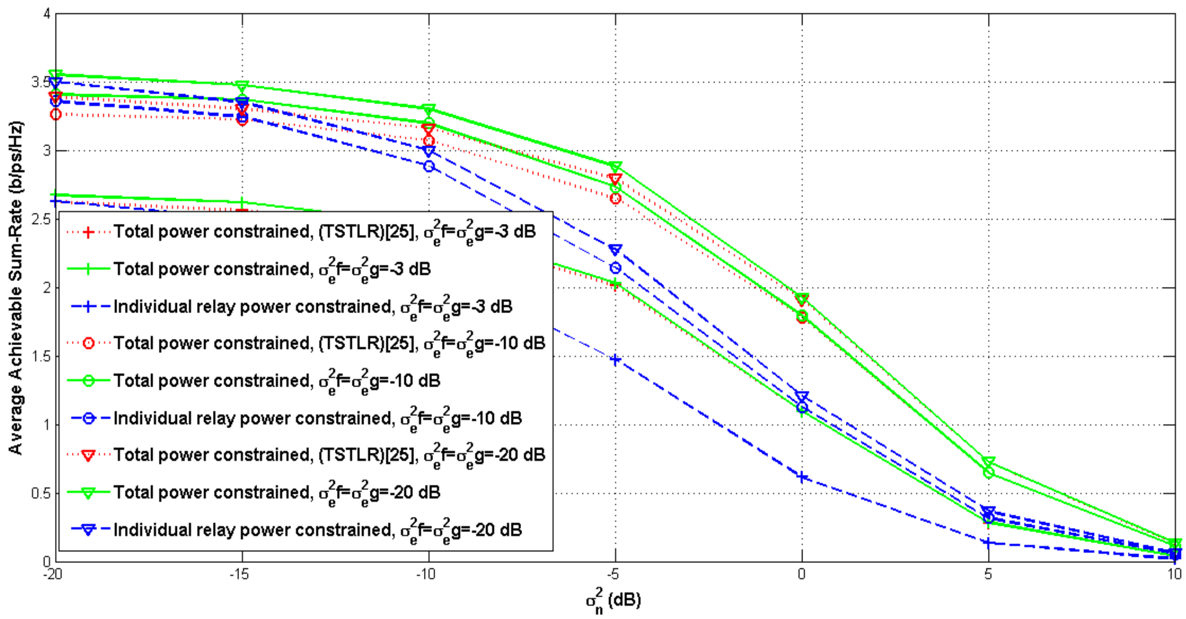

Fig 7. Average achieved sum-rate versus noise variance for imperfect CSI.

Simulation results showed that increasing the number of relays and the quality of uplink and downlink channels, and also decreasing the uncertainty of channels is the reason for increasing the achievable sum-rate. Although higher sum-rate can be achievable in total relay transmit power constraint scenario, the power consumption in different relays are approximately equal for individual relay power constraints scenario. It means that the received signals at destinations have the same quality which offers a higher diversity gain. In contrast, in total relay transmit power constraint scenario, meanwhile the summation of relay powers is lower than a predefined value some relays may consume very high or very low powers. Therefore, in this case, proper diversity cannot be accessed because some paths may experience very high or very low gains.

As the future works, sum-rate maximization problem considering multi-antenna nodes (source-destination pairs as well as relays), total and individual relay power constraints in a joint state can be investigated. Developing new methods to solve counterpart problem when destination nodes are mobile is the next problem. Finally, optimal relay node placement, in such a way that the achievable sum-rate is maximized may lead to finding the optimal number of relay nodes and achieving the best way to clustering the relay nodes.

APPENDX

Solving the optimization problem (34) is as the following procedure:

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Let $\mathbf{X} = \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$. We can write:

$$\begin{aligned} &\max \quad \frac{\mathrm{tr}\,(\mathbf{R}_{\mathrm{t}}\,\mathbf{X})}{\mathrm{tr}\,(\mathbf{H}_{\mathrm{t}}\,\mathbf{X}) + d\sigma_n^2} \\ &\text{subject to} \quad \mathbf{D}_{\mathrm{ii}}\,\mathbf{X}_{\mathbf{ii}} \le p_{x,i}^{max} \quad i = 1,2,\dots,R \\ &\mathrm{rank}\,(\mathbf{X}) = 1\,,\ \mathbf{X} \ge 0 \end{aligned} \tag{A-1}$$

or similarly, as follows [39]:

$$\begin{aligned} &\max \quad t \\ &\text{subject to} \quad \mathrm{tr}\left(\mathbf{X}(\mathbf{R}_{\mathrm{t}} - t\mathbf{H}_{\mathrm{t}})\right) \ge t\, d\sigma_n^2 \\ &\mathbf{D}_{\mathrm{ii}}\,\mathbf{X}_{\mathbf{ii}} \le p_{x,i}^{max} \quad i = 1,2,\dots,R \\ &\mathrm{rank}\,(\mathbf{X}) = 1\ ,\mathbf{X} \ge 0 \end{aligned} \tag{A-2}$$

This optimization problem is non-convex and may not be solved by a computationally efficient solution. In order to use semi definite relaxation technique, the optimization problem (A-2) can be changed to the problem (A-3) by ignoring rank constraint:

$$\begin{aligned} &\max \quad t \\ &\text{subject to} \quad \mathrm{tr}\left(\mathbf{X}(\mathbf{R}_{\mathrm{t}} - t\mathbf{H}_{\mathrm{t}})\right) \ge t\, d\sigma_n^2 \\ &\mathbf{D}_{\mathrm{ii}}\,\mathbf{X}_{\mathbf{ii}} \le p_{x,i}^{max} \quad i = 1,2,\dots,R \\ &\mathbf{X} \ge 0 \end{aligned} \tag{A-3}$$

As expected, the matrix $\mathbf{X}^*$ obtained by solving the problem (A1-3) may be rank one or not. If $\mathbf{X}^*$ is rank one, its solution is the optimal solution for (A-2).

It should be noted that the above-mentioned optimization problem is quasi-convex. It means that the feasible set for any t is convex. Supposing that t_{max} is the maximum value for problem (A1-3), if the convex feasibility problem [35] (A-4) is feasible (for any given t), then we have $t_{max} \ge t$. :

$$\begin{aligned} &\text{find} \quad \mathbf{X} \\ &\text{subject to} \quad \mathrm{tr}\left(\mathbf{X}(\mathbf{R}_{\mathrm{t}} - t\mathbf{H}_{\mathrm{t}})\right) \ge t\, d\sigma_n^2 \\ &\mathbf{D}_{\mathrm{ii}}\,\mathbf{X}_{\mathbf{ii}} \le p_{x,i}^{max} \quad i = 1,2,\dots,R \\ &\mathbf{X} \ge 0 \end{aligned} \tag{A-4}$$

In contrast, if the problem (A-4) is not feasible, $t_{max} < t$. Therefore, we can find t_{max} of problem (A-3) by solving the problem (A-4). Hence, a simple algorithm can be used to solve the quasi-convex optimization problem (A-3) at each step by using bisection technique.

Assuming the problem is feasible and starting with an interval $[l\ ,u]$ known to contain the optimal value t_{max}, we solve the problem at its midpoint $t = (l + u)/2$, to check that the optimal value is upper or lower than t. The interval will be updated accordingly to find a new interval. We set $l = t$ for feasible case and $u = t$ for non-feasible case and solve the problem (A-4) again.

It is clear that this approach takes more time with respect to two proposed schemes in this research. Thus, we propose to formulate three new design problems that have exactly the same constraints with sum-rate maximization problems but with better-behaved objective functions to find more-efficient solutions for maximizing sum-rate.

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