Abstract—In this paper, using adaptive networks-based fuzzy inference systems (ANFIS), a comprehensive closed-form solution for effective length of vertical grounding rod is extracted in such a way that the two effects of ionization and dispersion are simultaneously considered. In creating the model, training data are computed from multi-conductor transmission line (MTL). As a result, via ANFIS the effective length in such soils is efficiently computed. The simulation results show that considering both effects results in a length which is greater than the one in only-dispersive soil, and less than the one in only-ionized soil. Such a result is financially very important.

Index Terms—ANFIS, vertical rod, ionization, dispersion.

I. INTRODUCTION

Grounding systems such as vertical rods, horizontal electrodes, and grounding grids are conventionally used as tower-footing devices in order to discharge lightning current efficiently inside soil. To this end, grounding systems should be first analyzed and then effective length/area as defined in [1] should be computed. As known, these quantities are strongly dependent on frequency dependence of electrical parameters of soil, i.e., dispersion of soil [2-5], and soil ionization [6-8] when the towers are subjected to high-valued lightning strokes.

There is a number of research focusing on extracting effective length of dispersive grounding systems [4, 5], and ionized ones [7, 8]. But they consider either dispersion or ionization. To the best our knowledge, there is no formula for effective length of vertical rods considering both effects. Recently Jinliang He et al [9] have proposed a complex approach based on combining vector fitting (VF) [10] and method of moments (MoM) [11] to analyze grounding systems buried in dispersive and
ionized soils without extracting effective length/area. This motivates the authors to extract a formula for effective length of vertical rods considering both effects using adaptive networks-based fuzzy inference systems (ANFIS) [12]. In computing initial data in training process, instead of the hybrid approach in [9], the frequency domain approach proposed by J. L. Guardado [13, 14] is used. This approach was used in transient analysis of grounding system considering ionization of soil, but since it is in the frequency domain, dispersion of soil can be easily included. Recently Visacro et al [15] have used it for only-dispersive soil.

This paper is organized as follows. In section II, experimental models of ionization and dispersion of soil which are used in this study are presented. In Section III, neural networks based on radial basis functions are briefly explained. Section IV is focused on extracting effective length of vertical rod buried in dispersive and ionized soils. Finally concluding remarks are given in section V.

II. MODELING IONIZATION AND DISPERSION OF SOIL

As explained in previous section, two main factors namely ionization and dispersion of soil influence effective length/area of grounding systems. To include the two effects in transient analysis of power systems, they should be correctly modeled. These models are based on experimental results and have been used in a number of articles to investigate lightning performance of power systems [16]. In the next sub-sections, the two models are briefly explained.

A. Ionization of Soil

According to experimental tests carried out by researchers, when grounding systems are subjected to high-valued lightning current, electric field inside soil is increased in such a way that when it is greater than critical electric field of soil, resistivity of soil around grounding systems is considerably decreased (conductivity is increased). Since rod conductivity is very high, hence to model such phenomenon, it is assumed that the rod radius is gradually increased as shown in Fig. 1. Jiliang He et al [7] proposed a formula for grounding rods in only-ionized soils when lightning current is injected to its origin point as bellow

\[ \rho_{eff} = \frac{6.528(\rho T_M)^{0.379}}{I_M^{1.097}} \]  

In (1), \( I_M \) is maximum value of lightning current, and \( T_M \) is the time when the lightning current approaches to its maximum value.

B. Dispersion of Soil

The experimental results show that in some soils, electrical parameters of soil are frequency-dependent (dispersive soil) [2]. Hence a number of models have been proposed [2]. The simulation results show that such an effect considerably affects effective length especially when soil resistivity is
increased [3-5]. In this study, the dispersive model proposed by S. Visacro et al [3] is used. In this model, frequency variation of electrical parameters of soil is expressed as bellow

\[
\rho(f) = \rho_0 \left[ 1 + \left( 1.2 \times 10^{-6} \times \rho_0^{0.73} \right) \times (f - 100)^{0.65} \right]
\]

\[
\varepsilon_r(f) = \begin{cases} 
192.2 & f \leq 10\text{kHz} \\
1.3 + 7.6 \times 10^3 \times f^{-0.4} & f \geq 10\text{kHz}
\end{cases}
\]

Where \(\rho_0\) is low-frequency resistivity of the soil.

In such soils, S. Visacro extracted a formula for effective length [4] in only-dispersive soils as bellow

\[
L_{\text{eff}}(\rho_0) = \left( I_{c_0} - \beta \right)/\alpha
\]

Where

\[
I_{c_0} = -0.00086 \times \rho_0^{0.686} + 0.992
\]

\[
\alpha = 2 \times 10^{-3} + \exp(-1.55 \times \rho_0^{0.162})
\]

\[
\beta = -0.5 + \exp(-0.00046 \times \rho_0^{0.83})
\]

III. MULTI-CONDUCTOR TRANSMISSION LINE MODEL (MTL)

With reference to [13, 14], modeling grounding systems based on the MTL is summarized as bellow:

1-Take fast Fourier transformation (FFT) from lightning current to extract its frequency content.
2- Divide the electrode into N segments, and then represent each one as a two-port network as shown in Fig. 3. Parameters of these networks and the length of each segment, \( l_k \) are computed as bellow

\[
A = D = Y_0 \coth(\Psi l_k), \quad B = C = -Y_0 \csc h(\Psi l_k)
\]

\[
l_k \leq \lambda/10, \lambda = 2\pi/\beta_2, \beta_2 = \text{imag}\left(\sqrt{j\omega\mu(1/\rho) + j\omega\varepsilon}\right).
\]  

(8)

(9)

Where \( \Psi \) and \( Y_0 \) are propagation constant and characteristic admittance of each segment.

3- Compute sending voltage for the first segment at each sampled frequency (see (9) in [14]).

4- Compute leakage current for each segment, and then according to [7] compute electric field on the surface of electrode \( E_k \) as following

\[
E_k = \frac{I_{1k}/l_k}{2\pi(1/\rho f_m + j2\pi f_m \varepsilon)l_k}, \quad k = 1,2,...,N
\]

(10)

In (10), \( \varepsilon \) and \( \rho \) are dielectric constant and resistivity of soil respectively. \( f_m \) is a sample frequency from frequency content of the lightning current. \( l_k \) is the length of each segment. \( I_{1k} \) is the leakage current which is equal to the difference of sending and receiving currents of each segment.

5- If the computed electric field on the surface of each segment is greater than critical electric field of soil \( E_c \), then compute a new radius for each segment as following

\[
a_{\text{new},k} = a_k \frac{E_k}{E_c}, \quad k = 1,2,...,N
\]

(11)
Fig. 3. Representation of Fig. 1 as N segments and then modelling each one by a two-port network.

Fig. 4. Flow chart of the MTL as step by step in modeling ionization and dispersion of soil.

6- Repeat steps from 2 to 4 up to the electric field for each segment is less than critical electric field of soil.

7- With the new radius for each segment, compute sending voltage for the first segment at sampled frequencies.

8- Compute transient voltage via inverse fast Fourier transformation (IFFT).

9- Compute the effective length for the electrode based on the definition in [1].

At the first stage of the iteration process, each segment radius is assumed to be the original radius of the electrode and then it is increased up to $E_k < E_c$. Schematic diagram of the MTL is shown in Fig. 4.
IV. VALIDITY

In this section, the validity of the MTL in transient analysis of grounding electrode buried in ionized soil is investigated. To this aim, a horizontal electrode made of copper and length of 12m, diameter of 14mm, and buried in a soil with low-frequency resistivity of 42Ω.m was investigated. The same grounding system is analyzed by the MTL and compared with the measured results in [17]. The transient voltage with and without considering ionization is shown in Fig. 5. As it is seen in this figure, good agreement is achieved.

V. ADAPTIVE-NETWORK-BASED FUZZY INFERENCE SYSTEM (ANFIS)

In section II, the existing formulae for computing effective length considering either ionization or dispersion were introduced. Such formulae are created via curve-fitting techniques which needs too many input-output pairs especially when the number of inputs is increased. To remove this drawback, intelligent approaches such as conventional fuzzy inference systems (FIS) [19], neural networks (NN) [20], and adaptive network fuzzy inference systems (ANFIS) [12] can be used. Among these intelligent approaches, ANFIS is the most efficient. Hence, in this study, to extract a predicting formula for effective length in complex soils, ANFIS is used. Recently, it has been used in determining resonance frequency of different microstrip patch antennas [21]. Note that spatial membership functions [22-25] can also be used, but it is restricted to nonlinear systems with a few inputs and weakly nonlinear systems.

ANFIS is a class of adaptive networks that are functionally equivalent to fuzzy inference systems (FIS). The ANFIS architecture consists of fuzzy layer, product layer, normalized layer, defuzzy layer, and summation layer. A typical architecture of ANFIS for the problem under consideration consisting three inputs, and one output is depicted in Fig. 6, in which a circle indicates a fixed node, whereas a
Fig. 6. Schematic diagram of ANFIS for a three-input, one-output nonlinear system.

square indicates an adaptive node. As in Fig. 6, the output of ANFIS at each layer is computed as

**Layer 1.** In this layer, all crisp inputs are changed into linguistic inputs with linguistic values, i.e. small, medium and high. To this end, a number of membership functions for each input is first defined as bellow

\[
\mu(x) = \exp\left[-\left(\frac{x - c_i}{\sigma_i}\right)^2\right] \tag{12}
\]

where \(c_i, \sigma_i\) are parameters changing the shape of membership functions and defined in such a way that each membership function has a value of one at each sampled input and is smoothly decreasing at the neighbor sampled inputs. Variable ‘x’ is a symbol for each input.

**Layer 2.** In this layer, the firing strengthen for each rule is computed as bellow

\[
\mu_i = \mu_{A_i}(\rho)\mu_{B_i}(I_m)\mu_{C_i}(T_m), i = 1, 2, \ldots, N \tag{13}
\]

**Layer 3.** In this layer, the firing strengthen in the previous layer is normalized, that is

\[
\overline{w_i} = \frac{w_i}{w_1 + w_2 + w_3}, i = 1, 2, \ldots, N \tag{14}
\]

**Layer 4.** This layer is called fuzzy inference system (FIS) which all linguistic inputs are first changed into crisp inputs with crisp values, i.e. small, medium and high. This layer consists of a
number of if-then rules as bellow

Rule: if \( \rho_i \) is \( A_i \) and \( I_m \) is \( B_i \) and \( T_M \) is \( C_i \), then \( L_{eff} \) is \( L_{eff} = p_i \rho_i + q_i I_m + r_i T_M + k_i \)  \hspace{1cm} (15)

where \( A_i, i=1,2,..N \), \( B_i, i=1,2,..N \) and \( C_i, i=1,2,..N \) are the membership functions for inputs in the antecedent. Also \( p_i, q_i, r_i, k_i \) are constants determined using training data.

Layer 5. In this layer, all weighted outputs of FISs are combined, and finally the output for the problem under consideration is created as bellow

\[
L_{eff}(\rho_0, I_M, T_M) = \sum_{i=1}^{N} w_i z_i = \sum_{i=1}^{N} w_i (p_i \rho_0 + q_i I_M + r_i T_M + k_i) \hspace{1cm} (16)
\]

Using input-outputs pairs, training process is carried out up to predefined error (here root mean square error, MSE) is achieved. To achieve an acceptable error, optimization techniques can be used (here least square approach). The above relation can be used as comprehensively closed-form solution for effective length considering both ionization and dispersion of soil. Further information about ANFIS is found in [12].

VI. ANFIS-BASED MODEL FOR EFFECTIVE LENGTH

In this section with the use of the ANFIS model trained in the previous section, effective length of vertical rod (as ANFIS output) versus ANFIS inputs \((\rho_0, I_M, T_M)\) is computed. In the training process, 3 samples for \( \rho_0 \) (\( N = 3 \)) from interval of \([10 \ 1000] \Omega \cdot m\), 3 samples for \( I_M \) from interval of \([10 \ 200] \) kA, and 3 samples for \( T_M \) (\( N = 3 \)) from interval of \([2 \ 12] \) \( \mu s \) (\( N = 3 \)) are selected. Totally \( 3 \times 3 \times 3 = 27 \) input-output pairs or \( 27 \) if-then rules are used. The root mean square error (MSE) for the problem under consideration versus epoch is shown in Fig. 7. It shows that after about 100 epochs, the ANFIS is converged. The optimized membership functions are shown in Fig. 8, and the optimized output parameters are also listed in table 1. From now on, the effective length \( L_{eff} \) for arbitrary inputs \((\rho_0, I_M, T_M)\) which are different from training data is efficiently computed using relation (16). Fig. 9 depicts the effective length versus inputs in dispersive and ionized soils and it is also compared with the one in only-ionized and only-dispersive soils using relations (1) and (4) respectively. As seen in this figure, excellent agreement with the MTL is achieved. Comparison shows that considering both effects results in a value for effective length between situations of only-ionized soil and only-dispersive soil. Such results are was predictable because with reference to [7], ionization results in increasing effective length and dispersion results in decreasing effective length (for soils with low resistivity) [4]. Therefore, when two effects are considered the effective length should be a value between two cases. It is well known that this is financially of importance.
Fig. 7. Training error of versus epoch for the problem under consideration.

Table 1. Parameters of membership functions of $\rho_0$.

<table>
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<th>$\mu_\lambda$</th>
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<td>9.796</td>
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Table 2. Parameters of membership functions of $I_M$.

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Table 3. Parameters of membership functions of $T_M$.

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<td>$c_i$</td>
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<td>7.302</td>
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Fig. 8. Optimized membership functions ($\mu_A$, $\mu_B$, $\mu_C$) versus inputs ($\rho_b$, $I_M$, $T_M$) for the problem under consideration.

Table 4. Optimized parameters in the relation (16) when training process is converged.

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<tr>
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VII. CONCLUSION

In this study for the first time, using ANFIS a comprehensively predicting formula for effective length of grounding vertical rod was proposed. The results are interesting in such soils, so that the effective length is greater than the one in only-ionized soils and less than the one in only-dispersive soils. This makes it financially attractive to include the two effects simultaneously in complex soils.
Applying this approach for extracting effective area of grounding grids buried in such complex soils is another study that is in progress.

References


