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The Role of Regulatory in Price Control and Spectrum Allocation to Competing Wireless Access Networks

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***Abstract*—Withtherapid growth of wireless access networks, various providers offer their services using different technologies such as Wi-Fi, Wimax, 3G, 4G and so on. These networks compete for the scarce wireless spectrum. The spectrum is considered to be a scarce resource moderated bythespectrum allocation regulatory ("regulatory" for short) which is the governance bodyaiming to maximize the social welfare through moderation of the spectrum allocation table (SAT). In this paper, we present a three stage dynamic game model directed by the regulatory to maximize the clients' welfare. The regulatory controls the proposed prices announced by networks and it determinesthe tax in proportion to the price and load of each network. The model simulates the behavior of end users, network providers and the regulatory agent through which spectrum allocation strategy is deducted, the rules and parametersare defined, and the system equilibrium in terms of resource allocation and pricing is analyzed. The experimental results show that the proposed spectrum allocation schema results in a situation with the highest clients' welfare and network providers have enough advantages to stay in the market.**

***Index Terms*-**Dynamic Pricing,Game theory,Heterogeneous wireless networks, Nash equilibrium, Spectrum Allocation.

I. INTRODUCTION

The existence ofvarious wireless technologies and different service providers from one side and the growth of multi modal smart devices that are able to connect to different available networks simultaneouslyfrom the other side generate an environment named heterogeneous wireless access network (HWAN). The concept of '*Always Best Connected'* [1], means that the clients with multi modal devices like to receive their service from the network provider that increases their payoff and a client is smartenough to select the best network. The payoff of clients may differ according to their application requirements, device state and the manner of the client. According to the three important profiles, 1)Application profile, 2) Device profile and 3) The client profile, the available networks are ranked and weighted by the clients.

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One of the most important decisions for the network provider is the pricing mechanism. The network providers have to monitor the state of the network, continuously. The number of clients, total delivered bandwidth, etc. is the acquired data from the current state of the network and then the price and the amount of prepared bandwidth should be adjusted. Through pricing mechanism, a networkprovider likes to increase its payoff by increasing the price while it knows that clients like to receive (buy) the bandwidthat minimum charge.

The process of resource allocation and pricing in HWAN is the same as the oligopolymarket[2], the network providers prepare the services with different Quality of Service(QoS) which is similar to commodities with small differences that are produced by different firms in an oligopoly market. The network providers offer the best response according to theclients' total request and the cost of preparing the service.

Rather than clients and network providers we have to consider the important role of the regulatory as the government agent in the process of spectrum allocation. The network provider should be licensed by the regulatory and the regulatory charges the network providers according to their spectrum use. Also, the regulatory can apply the proper taxation mechanism to control the price of the service of the networks. One of the most used strategies for the regulatory to distribute the total spectrum within wireless network providers is the auction mechanism[3, 4], but in auction the utility of winning operators is maximized and it does not guarantee an increase in the clients satisfaction. It is telling that the Regulatory shouldnot only interact just with the network operators, but also it should consider theimpact on the clients.Such multi-level interactions can bemodeled by a three-tier model.In [5] a three-tiermodel (the Regulatory, the network operators, and the end users) is proposed where the impact of the regulatory spectrum allocation on the clients' satisfaction as well as the regulatory income, are studied. The main contribution of thispaper is to fill this important policy gap.

Our main contributions in this work are as follows:

- *Applying the Taxation rules according to the overcharging networks*. The Regulatory applies a mechanism that controls the price of service and it is not profitable for network providers to overcharge.
- *Spectrum allocation taking clients' welfare into consideration*.The regulatory allocates the spectrum to the networks in such a waythat the clients' welfare is increased.
- *Clients are weighting the networks according to their current state-vector*. Clients are weighting and ranking the available networks and preparing their request vector. As the utility function of clients is concave, one client can receive its service from different available networks.
- *Increase the number of online clients*. According to the defined criterion for the clients' welfare that is the ratio of the total released bandwidth to the total paid amount, when this

criterion increases it means that with a specific amount of money the released service is increased and more clients can benefit from network connectivity.

A. *Related Work*

The oligopoly markets model is used in Niyato and Hossain in [6] proposed model. They suggested a leader-follower competitive game model within providers to find the optimum prices. The Auction based mechanism proposed bySallent et al., in[7], that users periodically bidon the amount ofservice, price and the QoS requirement for the provider. Then, the service providerdecides on the resource allocation that maximizes itsrevenue. To approve this multi-unit sealed-bid auction, they used a manageragent that facilitates negotiation between a mobile user and a serviceprovider.In some other works, the authors are focused on the unlicensed or free spectrum like Wi-Fi and suggest approaches for pricing and collaboration of different providers.For example Duan et al. in [8] studied the mechanism of pricing in global Wi-Fi, whereas many Wi-Fi providers offer high performance mobilecommunication experiences (e.g.,AT&T in the US, BT Openzone in the UK) and are deploying a large number of Wi-Fi APs in theirlocal markets. They suggest a two-stage Stackelberg game between provider i and a group of N_i local users to find the price and the amount of released service. Finally the equilibrium price is calculated locally and the welfare of clients increases when such usagebasedpricing is employed. Again Duan et al. in [9]proposed an extended model.In previous model local providers were ignored while in the new suggested pricing model, local Wi-Fiproviders exist and they tend to cooperate with global Wi-Fi providers and give up part of their income to global Wi-Fi providers.Matinkhah and Khorsandi in [10], proposed a user-centric mechanism for bitrate allocation to balance the load of networks. In this work they did not consider the client budget constraint in network selection. They didn’t address or model clients’ behavior and their role in network resource allocation.Cao et al. in [11], propose a dynamic bidding game-based spectrum allocation model for heterogeneous wireless service in cognitive radio networks. They formulate the allocation decision-making process as a dynamic bidding game, analyze both spectrum owners (Primary users) (PUs) and the spectrum buyer (Secondary users) (SUs) utility functions.

Our paper is different from previous works: 1) we try to maximize the clients welfare but in other works maximizing the utility of networks is the goal; 2) we used a three-level game model, where the regulatory agent in the first level guides the equilibrium to the suitable position and controls the price proposed by networks;3) we find the suitable spectrum allocation for each operator that maximizes the clients welfare 4)Price controlling by the Regulatory prevents price rises as unfair.

II. SYSTEM MODEL

In the following a game model is presented and an action plan for all agents to find theϑ_j^*, $x_{i,j}^*$ and p_j^*that maximizes the agents’ utilities. Where ϑ_j^* is the best allocated spectrum to the network j,

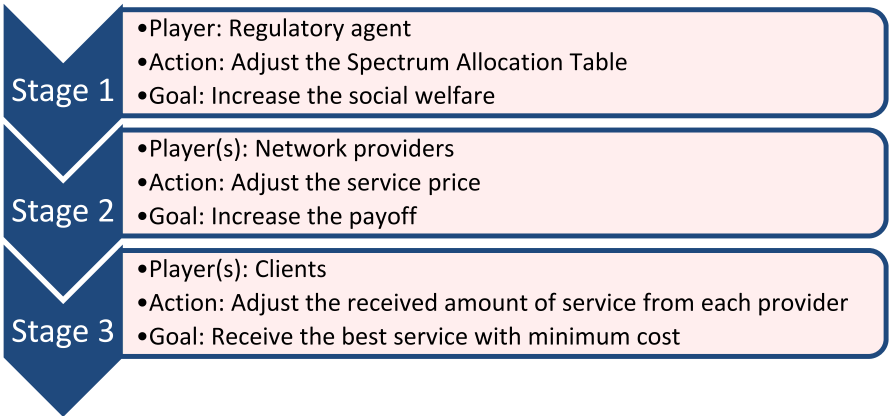


Fig.1. Three stages game in one iteration of repeated decisions

$x^*_{i,j}$denotes the best bitrate allocated to the client$i$ by the network j and p^*_j stands for the best price of a unit of bitrate offered by the network j.

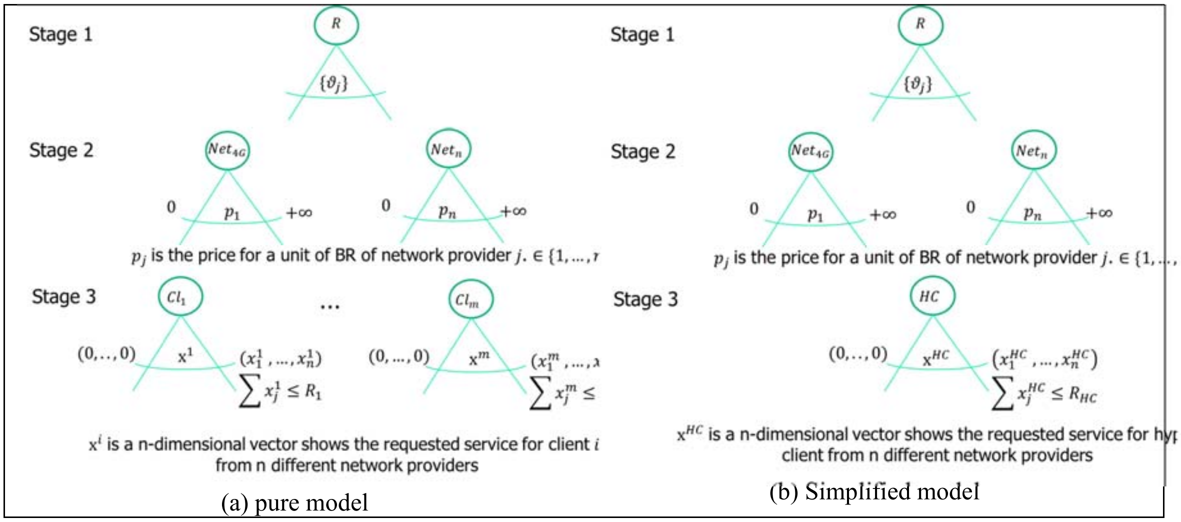
A. *Solution Overview*

Most researchers use the aggregate utility of all clients as the welfare of the clients (e.g. [5, 6, 7]) but one of the most used criterions in microeconomics is the purchasing power. When the purchasing power in a society increases it means that peoples can buy their required materials with lower cost[12].

It is brief that clients can only control and manage their request bundle according to their budgets, whereas in game theories such agents are named followers. The leader of the game plays the important role. In HWAN, if we weaken the role of the regulatory, the network operators will be the leaders and they may increase the prices while it is profitable for them. The proposed model tries to control the offered price by the network providers to increase the purchasingpower of the clients.

So, the regulatory should use the provided factors to guide the equilibrium of the market toward its goal. In proposed model, the decisions in the HWAN as repeated events; our approach uses a three-stage game in iteration, with regard to the independence of the iterations.As depicted in **Error! Reference source not found.**, in the first stage, the regulatory adjusts the taxation parameters and the certificates of different networks to use the spectrumand in the second stage the network operators decide on the price of a unit of service (the bitrate in our model), in the third stage clients decide on the amount of service requested from different providers. Note that the client decisions, the network decisions and the government decisions have different time scales. Clients decision happen in short time slots (e.g. each one hour) and the network decisions happen in the middle time slots (e.g. one week) and the government decisions happen in long time slots (e.g. 6 months).

The extensive form of our proposed game is showed in Fig.2(a). To reduce the complexity of the model the action of m clients in the third stage is replaced with a single player named hyper client (Fig.2(b)). In the first stage the regulatory adjusts the Spectrum Allocation table (SAT). The



(a) pure model (b) Simplified model

Fig.2. Extensive game model for the proposed three stages game

parameter$C_j$is the charge of network j for the license of using a unit of spectrum. The SAT determines the spectrum allocated to each network provider ($\{\vartheta_j\}$). The parameter p_j in stage 2 denotes the price of a unit of service (especially bitrate) for the network j, while the parameter x^i denotes the request bundle of client i from two different available networks.

The single client private knowledge are w_j^i, h^i, r^i which are respectively the weight of network j, the budget of the client i and the required bitrate of client i. The hyper client private knowledge are W_j, H, R that are respectively the expectation of clients' weight of network j, total budget and the total required bitrate. The relation of these parameters is:

$$H = \sum_{i=1}^{m} h^i\,, R = \sum_{i=1}^{m} r^i\,, W_j = E\left(w_j^i\right) = \frac{1}{m}\sum_{i=1}^{m} w_j^i. \tag{1}$$

B. *Clients Request bundle*

Clients are the agents that act in the final stage and the strategy of a client is creating a request bundle to maximize its payoff. Hence the request bundle of client $i$in mixed strategy is (x_{4g}^i, x_{3g}^i).Suppose $u^i(x^i) = H^i - \sqrt[r]{\sum_j \left(w_j^i . x_j^i . p_j . \theta_j\right)^r}$, is the client utility function. All clients solve the following optimization problem to prepare their request bundle:

$$\begin{aligned} &\mathit{Maximize}\, u^i\left(w^i, x^i, p, \theta_j\right) \\ &s.t. \\ &1)\, p.x^i \le H^i \\ &2)\, x^i \in \mathbb{R}_+^n \end{aligned} \tag{2}$$

3) $\sum_j x_j^i \geq R^i$.

Where R^i, H^i is the required service and total budget of client i, respectively. Note that $i\epsilon\{1,\dots,m\}$

C. Total clients' response vs. hyper clients' action

In simplified model,m independent rational clients are replaced by a single rational player named hyper client. Suppose $(\{W_j\}, H, R)$ are hyper client private information and $(\{w_j^i\}, h_j, r_j)$ is the private information of client i. Suppose the weight,budget and required bitrate of clients have Gaussiannormal distributions that are respectively $\overline{w}_j \sim \mathcal{N}(\mu_{w,j}, \sigma_{w,j}^2))$, $\overline{h} \sim \mathcal{N}(\mu_h, \sigma_h^2))$ and $\bar{r} \sim \mathcal{N}(\mu_r, \sigma_r^2))$, hence according to the mixed strategy of client i, x_j^i shows the amount of bitrate requested by client i from network j (solution of (2)). Consequently total bitrate requested from network j is χ_j and it is the product of two independent normal distributions ($\chi_j = \sum x_j^i$); Many researchers studied the specifications of the production of two normal distributions and it is brief that in general the distribution of two normal distributed random variables is not normal[13, 14]; but for simplicity with some relaxations such as $\sigma_r^2 \sim 0$we can write $\chi_j \sim \mathcal{N}(\mu_{\chi,j}, {\mu_r}^2 \sigma_{\chi,j}^2))$ where $\mu_{\chi,j} = \mu_{w,j}\mu_r$ and $\sigma_{\chi,j}^2 \cong \sigma_{w,j}^2$. The solution of problem for hyper client player isX_j has a normal distribution. According to our relaxation the distribution of X_j is near the distribution of χ_j.

We have assumed 'hyper client' to be able to simultaneously subscribe to all networks. However, 'hyper client' represents a large group of similar 'simple' clients. The simple clients does not necessarily have the capability to simultaneously use different service providers. It means that, some of the 'simple' clients consisting a hyper client may subscribe toonly one network. But the behavior of hyper client is the same as overall response of all clients.

D. Networks'Response

The income of a network provider is the product of the bitrate price and the amount of the sold service ($p_j . L_j$, $L_j = \sum_i x_j^i$), and the cost of network consists is the charge for the license of using the assigned spectrum. Suppose π^jdenotes the utility of network j then $\pi^j = p_j L_j - \phi_j(\vartheta_j) - Tax(L_j)$; where ϕ_j is the spectrum license charge and $Tax(L_j)$ is the tax for sold service that network j should pay to the Regulatory.

Suppose L_j^{max} denotes the maximum releasable bitrate (maximum load) for network j, hence, $L_j^{max} = \eta_j \vartheta_j$; where η_j is the technology efficiency. For example, the efficiency of 4G which used LTE is four times better than 3G that uses WCDMA technologyper carrier [15].

E. Regulatory Strategy

Suppose Γ is the total available spectrum that regulatory is going to distribute within networks ($\{\vartheta_j\}$). In the following it is showed that the Regulatory decision on SAT and β directly effects on the clients' welfare. In this paper, adjustment of freedom parameterβ is not concerned but our approach for SAT adjustment is presented in the following section. Generally in our model, the Utility of Regulatory is defined as follow:

$$U_r(\lambda,\beta) = CWF + \beta.RI \tag{3}$$

Where CWF is the clients' welfare and RI is the regulatory income.

III. STRATEGY OF AGENTS IN DIFFERENT STAGES OF THE GAME

In this section the strategy of players and the effect of each agent's action on other agents that introduced in **Error! Reference source not found.**is described.

A. *Stage 1: The Regulatory Strategy*

There are two functions $\phi(.)$ and $Tax(.)$related to license of use the spectrum and the Tax of sold service. The parameters and the structure of these functionsare defined by the regulatory. The following forms for these functions are considered:

$$\phi(\vartheta_j) = p_s\vartheta_j \tag{4}$$

$$Tax(L_j,p_j) = \frac{(p_j - p_0)^2}{2\beta}(L_j)^{2/3}. \tag{5}$$

Wherep_s the license fee of a unit of spectrum is, ϑ_j is the allocated spectrum to network j andβ is the freedom parameter that specifies the amount of networks freedom for pricing.If a network provider offers a high value for its service it should pay more tax and below,when the best response of the networks is analyzed, the specifications of the proposed tax function will be discussed in more detail.

B. *Stage 2: Networks' Best Response (Networks Price Tuning)*

The utility function of network provider j is:

$$\pi^j(L_j,p_j) = p_jL_j - p_s\vartheta_j - \frac{(p - p_0)^2}{2\beta}(L_j)^{2/3}. \tag{6}$$

The network providers try to maximize their payoffs. Note that, in pricing approach in most models the competition is between network providers, and generally the best response of a network is calculated according to the action of other networks. But in proposed approach it is assumed that the payoff of a network is controlled by the regulatory and networks are not as free to offer a price only according to the request of the market and the constraints of the regulatory are more restricted than the offered price by other network providers because our approach in this research increased the power of regulatory to control the market itself. The network provider finds the best price that maximizes $\pi^j(L_j,p_j)$ the same as that proposed in the following:

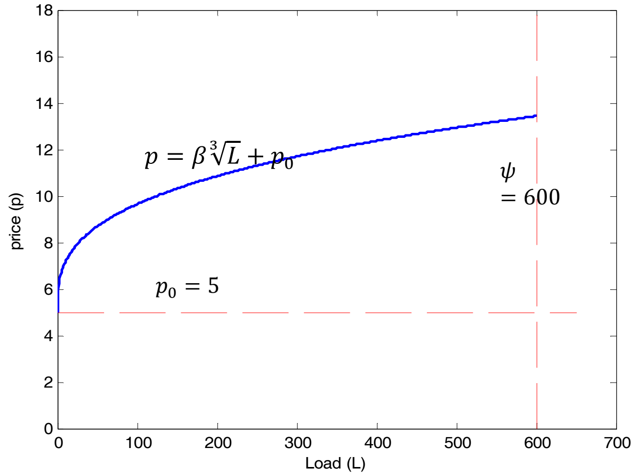


Fig.3. Best Response of the network according to the load (L) and parameters that have been determined by the regulatory (ψ, p_0, β).

$$\frac{\partial}{\partial p_j}\pi^j(L_j,p_j) = L_j - \frac{\partial}{\partial p_j}\left(\frac{(p_j-p_0)^2}{2\beta}(L_j)^{2/3}\right)$$

$$\frac{\partial}{\partial p_j}\pi^j(L_j,p_j) = 0 \xrightarrow{L_j>0} L_j - \frac{p_j-p_0}{\beta}(L_j)^{2/3} = 0 \qquad (7)$$

$$p_j = \beta\sqrt[3]{L_j} + p_0.$$

Suppose ψ_j is the maximum load of network j which is determined by the technology efficiency used by network j ($\psi_j = \eta_j\vartheta_j$). The curve of network best response is drawn in Fig.3. Whenever βvalue is higher, the slope of the curve increases and when the load of network increases the network offers a high price even though it should pay more tax.

C. Stage 3: The clients response (Request bundle)

Clients prepare their request bundle according to their preferences and their decisions are made independently and simultaneously. All clients solve the optimization problem (2)and they find the best request bundle to maximize the utility u^i. The proposed utility function (Fig.4) is concave and differentiable.

Each one of the constraints of the problem (2) denotes a hyper-plane in the request space, and the scope of result of optimization problem is the intersection of these constraints (Fig. 5). In some conditions the intersection of these areas may be empty, then our problem will not have any answer.

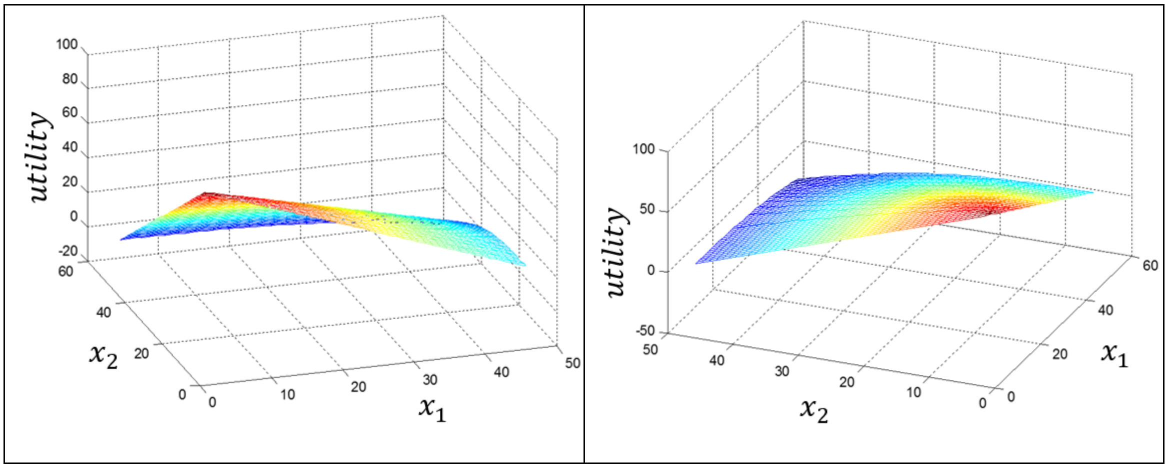


Fig.4. The concave surface of utility function for a client with $H^i = 90, w^i = (1,1), \theta = (1,1), p = (7,9)$

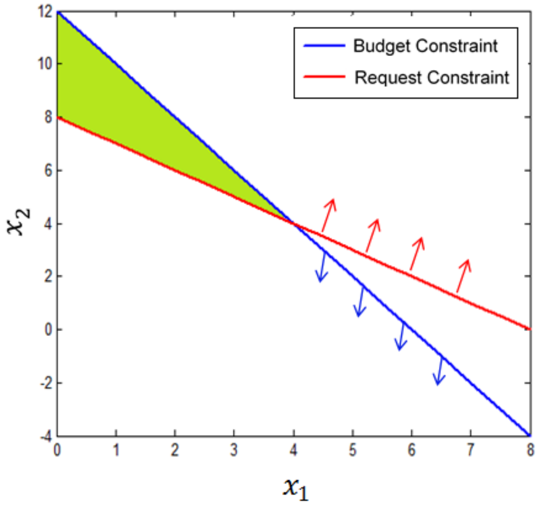


Fig. 5. The scope of result for the problem(2) is restricted by the constraints.

Theorem 1- The clients' response is unique. Suppose $\left(p_j\right)_{j\in\mathcal{J}}$ is the offered price vector of n existing networks and $\{w^i\}_{i\in\mathcal{I}}$ is the weight vector of clients and also $\left(\theta_j\right)_{j\in\mathcal{J}}$is the QoS coefficient of networks. Note that $\mathcal{I} = \{1, \dots, m\}$ is the index set of m existing clients and $\mathcal{J} = \{1, \dots, n\}$ is the index set of networks. The function $x^i\left(w^i, p, \theta, H^i\right)$ (the problem(2)) which can be briefly write as$x^i\left(p, H^i\right)$ why other parameters are not in the action list of any agents,has a unique answer for each price vector p.

Proof:

The objective function is a concave function and the intersection of the constraints is a convex area. After framing the Lagrangian equalities, we acquire:

$$\mathcal{L}(x^i,\lambda) = u^i(x^i) - \lambda_1[p.x^i - H^i] - \lambda_2[R^i - 1.x^i]. \tag{8}$$

Where 1 stands as a n-dimensional vector in which all elements are 1. According to the Kuhn-Tucker theorem[16],if $x^{i*} \gg \mathbf{0}$ solves(8), then there exists a $\lambda^* \geq 0$ suchthat (x^{i*},λ^*)fulfill the followingKuhn-Tucker conditions:

$$\frac{\partial \mathcal{L}}{\partial x_j^i} = \frac{\partial u^i(x^{i*})}{\partial x_j^i} - \lambda_1 p_j + \lambda_2 = 0, \quad j = 1, \dots, n \tag{9}$$

$$p.x^{i*} - H^i \leq \mathbf{0} \tag{10}$$

$$\lambda_1^*[p.x^{i*} - H^i] = 0. \tag{11}$$

$$R^i - 1.x^i \leq \mathbf{0} \tag{12}$$

$$\lambda_2^*[R^i - 1.x^i] = 0. \tag{13}$$

So x^{i*} is the unique request bundle for the client i.

□

Using the Theorem 1, the load of each network which is a combination of the requests from the network provider ($L_j = \sum_{i\in\mathcal{J}} x_j^i$) is unique and it could be considered as the ***Best Response*** of allclients ($L_j = BR_c(p_j)$),the $L_j$is defined as the best response of the Hyper client.

IV. NASH EQUILIBRIUM

A. *Nash Equilibrium (NE) and Subgame Perfect Equilibrium (SPE)*

The concept of Nash Equilibrium and subgame perfect equilibrium can be described briefly as follows [27]:

Definition 1 (Nash Equilibrium).*Suppose a game* $\{\mathcal{J}, (S_i)_{i\in\mathcal{J}}, (u_i)_{i\in\mathcal{J}}\}$, *where* $\mathcal{J}$ *is the set of players,* S_i *is the strategy set of player* $i \in \mathcal{J}$ ($S_i = \{s_i\}$), *and* u_i *is the utility of player* $i \in \mathcal{J}$. *The notation* $s = (s_i, s_{-i})$ *is used to show a strategy profile of all users, where* s_i *is the strategy profile of player iand* s_{-i} *includes the strategy choices of all players other than* i. *Let* $S = \prod_i S_i$ *as the set of all strategy* *profiles. A strategy profile* $s^* \in S$ *is a (pure) Nash Equilibrium if and only if* $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$ *for all* $i \in \mathcal{I}$.

Our proposed game is a dynamic game, the players act sequentially in different stages with different time slots. We need to introduce the subgame concept which is part of a dynamic game, and in our solution there are three subgames.

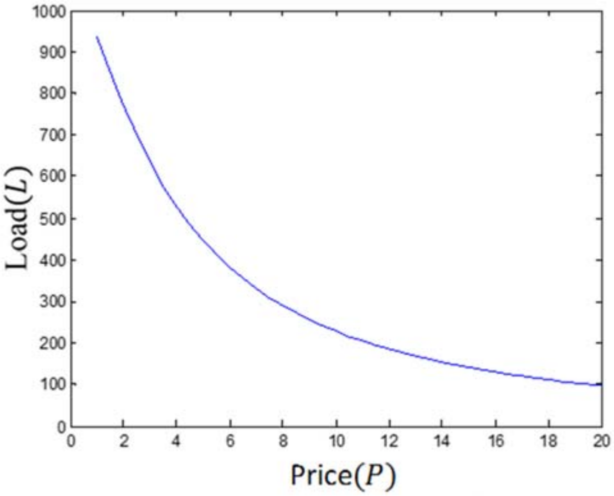


Fig.6. Response of hyper client to the offered price (Demand Curve)

Definition 2 (Subgame Perfect Nash Equilibrium).*For a game* $\{\mathcal{I}, (S_i)_{i\in\mathcal{I}}, (u_i)_{i\in\mathcal{I}}\}$, *a strategy profile* $s^* \in S$ *is a SPNE if and only if for all subgames of the game the strategy profile* s^**is a NE.*

A strategy profile of our three-stage game is an SPE if the choices of the Regulatory, the network operators, and the clients constitute a Nash Equilibrium in each of the subgames of the whole game. In other words, no player at SPNE will deviate unilaterally from his equilibrium strategy.

One of the best ways to analyze a dynamic game is the *Backward Induction*. We start to solve the game in the smallest (or final) subgame and after finding the NE of that game we come further in the tree until we meet the root.

B. Nash Equilibrium of Subgame (Stage 2 and 3)

1) Price and service allocation equilibrium

The response of all clients together generates the load of the networks so we assume the summation of clients' request bundle as a single rational agent named **hyper client**. Thus, we find the best response of the clients to the service prices (Demand Curve) and the best response of networks to the load of the network (Supply Curve). The network best response has a closed relation form and it is drawn in Fig.3, but unfortunately the clients' best response does not have a closed relational form.But the clients' best response curve as depicted in Fig.6 has special characteristics; it is positive, continuous, decreasing.

Hyper client and networks rationally follow the best response suggestion and finally there exists only one equilibrium point wherein the best response curves meet each other in other word the intersection of Demand curve and supply curve is the Nash equilibrium (Fig.7).

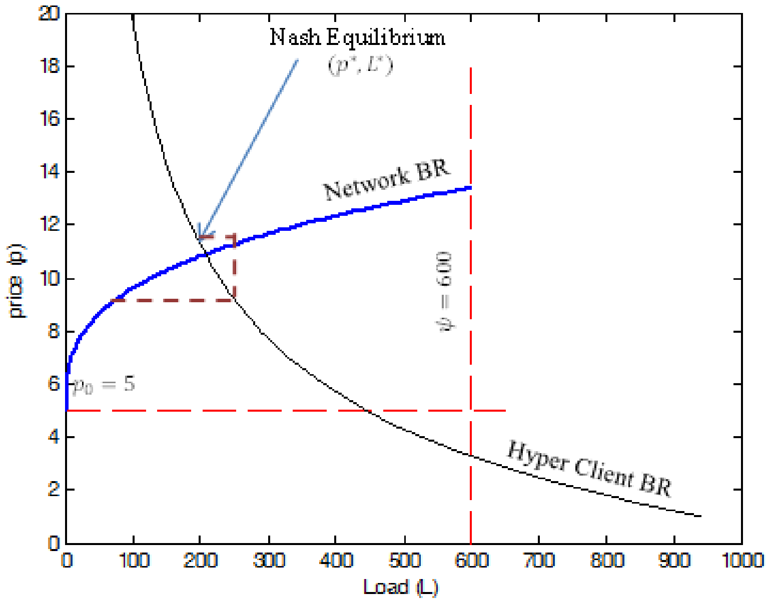


Fig.7.Convergence to the Nash Equilibrium for price and load from the intersection of Best Responses

Note that although the network best response curve is static, the hyper client BR curve depends on other network prices rather than current network price. But during the convergence process when all network providers are tuning their price, we see that this iteration converges to a single point[1].

The most important key in the price equilibrium is that the intersection of these curves is only one point.

2) *Impact of QoS on the Demand Curve*

To enlighten the importance of the QoS factor, suppose the condition of an environment whose equilibrium state is shown in Fig.7 but the bitrate constraint of such network is 150 ($\psi = 150$). Hence, the equilibrium is out of network access! In this situation the QoS of the network decreases until the equilibrium is located inside the defined network limited service (bitrate). Note that $u^i(x^i) = H^i - \sqrt[r]{\sum_j \left(w_j^i . x_j^i . p_j . \theta_j\right)^r}$ in problem(2); so, the parameter θ_j has a direct effect on the client's utility and their decisions.

The parameter θ is the inverse of quality of service ($\theta = 1/QoS$) and the maximum value of θ is 1. Assume that the regulatory limits the network service (bitrate) to $\psi = 450$ and the equilibrium point

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[1] Actually the hyper client player's best response BR_{HC} is a function of all networks prices ($BR_{HC} = X(p_j, p_{-j})$) and in the drawn figure we suppose that other networks did not change their price while in real world all networks tune their prices simultaneously.

is located outside of the network service scope, the QoS decreases until the released service is equal to the network service threshold. As depicted in Fig.9 the Nash equilibrium when $\theta = 1$ is located outside of the defined threshold by the regulatory; in this case, the QoS reduces from 1 to 0.6 where the $L^{*} = 450$.

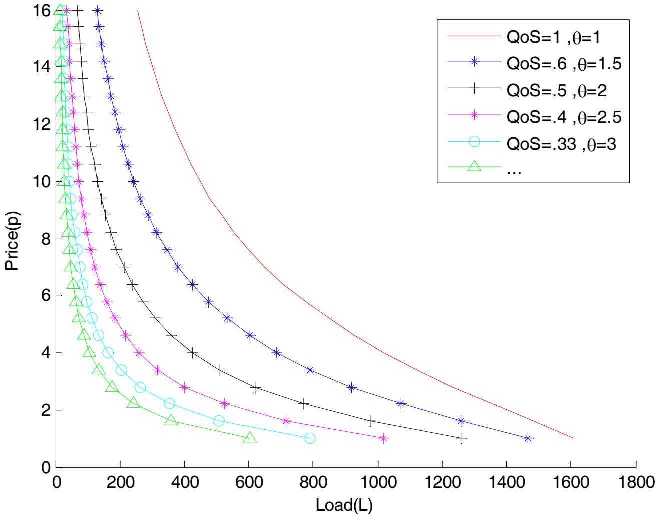


Fig.8. The impact of QoS on the hyper client response curve

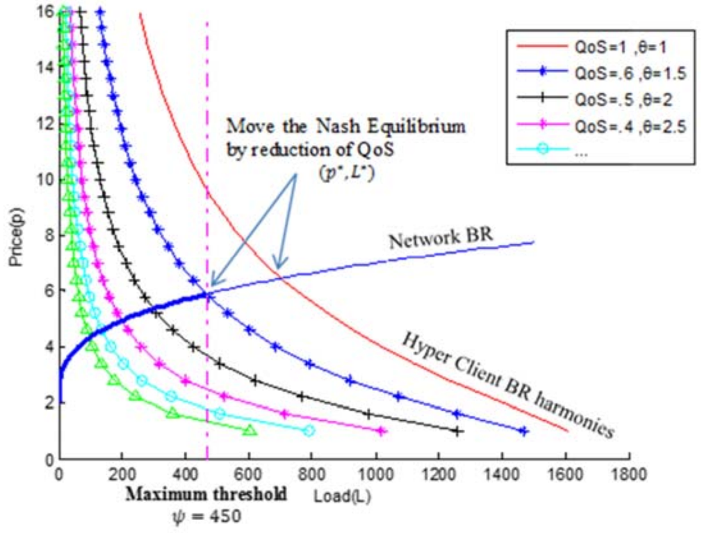


Fig.9. Harmonies of hyper client BR according to different values for θ. Reduction of QoS to meet the maximum threshold service assigned to the network.

The network provider controls the load by adjustment of QoS using the factor θ. Whenever the equilibrium point is located out of the defined service constraint then the network reduces the QoS (increases the θ) to reduce its load. The strategy of the network to adjust the factor $\theta$is as follows:

$$\theta_j^{new} = \begin{cases} \theta_j^{old} & L_j = \psi_j \\ \min\left(\theta_j^{new} + \hat{\theta}, \theta_0\right) & L_j > \psi_j \\ \max\left(\theta_j^{new} - \hat{\theta}, 1\right) & L_j < \psi_j. \end{cases} \tag{14}$$

As noted before, the price is adjusted in the middle term time slots and the client response is done in short term time slots. The same as the client response, the factor $\theta$is adjustedin short term time slots. The value of $\hat{\theta}$ could be a static positive value and it could also be a function of $(\psi_j - L_j)$, but in our simulation we considered a small positive value for $\hat{\theta}$. The constant value $\theta_0$defines the maximum value for the θwhich is 3 in our simulations.

C. *Direction of system by the Regulatory in Stage 1*

We formulate the client and network decisions and their reactions in previous sections. Hitherto, network providers and clients are playing according to the designed mechanism by the regulatory. We know that the regulatory defines three important parameters $\{p_0, \beta, (\psi_j)_{j\in\mathcal{J}}\}$:

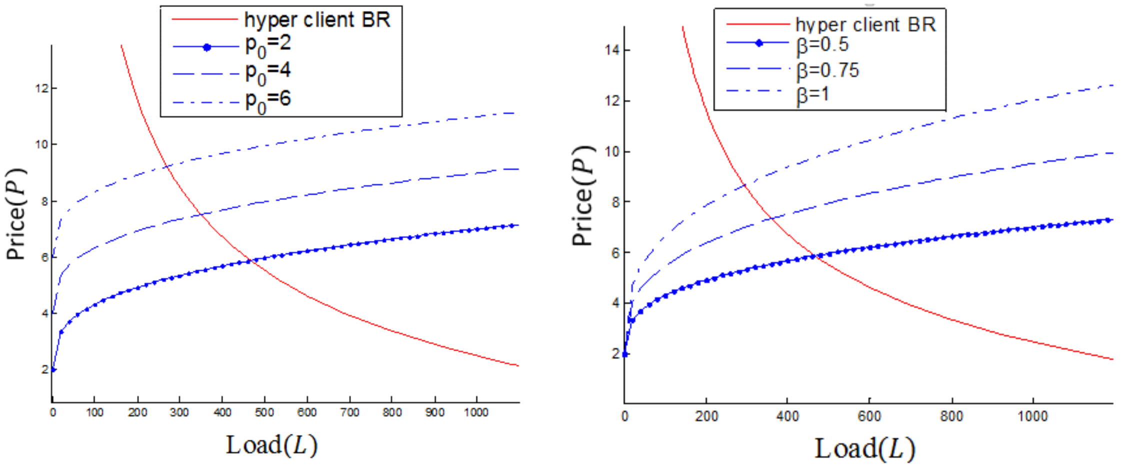
The parameter p_0 is the price of a unit of spectrum. If the regulatory increases the p_0, its income increases but the network best response curve in Fig.10(a)moves up and then its intersection with the hyper client response curves is located in a position with a lower amount of load. In other words, the increase of p_0 results in the reduction of the clients' welfare.

The parameter β controls the offered price by the networks. As depicted in Fig.10(b), if the regulatory increases the β, it allows the networks to offer higher charges for their service (bitrate), and increase of β results in decrease of clients' welfare.

Using the spectrum allocation table $\{\vartheta_j | j \in \mathcal{J}\}$, the regulatory controls the maximum bitrate of network providers. The most important role of the regulatory is the adjustment of spectrum allocation table which is done in long term time slots. Suppose the parameter $\mathcal{W}$ shows the clients' welfare which is defined as the division of total delivered service to the total paid amount by the clients. As noted before, for simplicity we use the maximum bit rate allocation table $\{\psi_j | j \in \mathcal{J}\}$ instead of spectrum allocation table:

$$\mathcal{W}(\vartheta_j)_{j\in\mathcal{J}} = \frac{\sum_{j\in\mathcal{J}} L_j}{\sum_{j\in\mathcal{J}} L_j p_j} \tag{15}$$

Suppose two different bitrate allocation profiles $\dot{\vartheta}$, ϑ' where $\mathcal{W}(\dot{\vartheta}) > W(\vartheta')$, this means that clients spend less cost to receive the same service in allocation profile $\dot{\vartheta}$ rather than ϑ'; so the allocation profile $\dot{\vartheta}$ is much better for the clients. Note that we suppose equal charge (p_0) for a unit of spectrum and the income of the regulatory does not change for different allocation profiles and the main constraint of the regulatory is maximizing the clients' welfare. Another constraint of the regulatory is satisfying the minimum assignable spectrum ($\tilde{\vartheta}$) to each network provider. So the regulatory solves the following optimization problem:



a) The impact of the $p_0$on the network best response curves

b) The impact of the freedom parameter (β) on the network best response curves

Fig.10. The impact of parametersp_0 and βon the network best response curves.

$$\begin{aligned} & \textit{Maximize}\, \mathcal{W}(\vartheta) \\ & \textit{s.t.} \\ & \textit{1)}\, \forall j \in \mathcal{J} \quad \vartheta_j \geq \tilde{\vartheta}_j \\ & \textit{2)} \textstyle\sum_j \vartheta_j \leq \Lambda \end{aligned} \tag{16}$$

The impact of parameter ϑ_j in the welfare relation (15) is not linear because both the load and the price are dependent on the maximum allocated bitrate to each network. Our solution is using a mechanism to increase the current welfare. In other words, we use a sequence generator function to improve the clients' welfare.

$$\begin{gathered} \bar{p} = \frac{\sum_{j\in\mathcal{J}} L_j p_j}{\sum_{j\in\mathcal{J}} L_j} \\ \lambda_j = \begin{cases} \vartheta_j + \delta - \tilde{\vartheta}_j & p_j < \bar{p} \\ \max(0, \vartheta_j - \delta - \tilde{\vartheta}_j) & \bar{p} < p_j \\ \vartheta_j - \tilde{\vartheta}_j & p_j = \bar{p}. \end{cases} \\ \vartheta_j = \tilde{\vartheta}_j + \left(\Lambda - \sum_{j\in\mathcal{J}} \tilde{\vartheta}_j\right)\left(\frac{\lambda_j}{\sum_{j\in\mathcal{J}} \lambda_j}\right) \end{gathered} \tag{17}$$

This strategy guarantees the minimum assignable spectrum to each network and the remaining spectrum is distributed within networks according to their offered price toward increasing the purchasing power of the clients or the clients' welfare. This mechanism tries to minimize the average price of the service.

V. SIMULATION RESULTS

In our simulation we want to trace the effect of regulatory decisions on the networks income as well as the clients' welfare.

Consider a HWAN with three Network providers and 50 clients. The clients' budget for a unit of service has normal distribution $H^i = N(\mu_H, \sigma_H)$ where we used $\mu_H = 10$and $\sigma_H = 2$. Also, the required service (bitrate) of clients has normal distribution $R^i = N(\mu_R, \sigma_R)$ where we used $\mu_R = 20$ and $\sigma_R = 3$.The network weights for each client are generated by a uniform distribution in [0,1] domain and the summation of weights is normal to 2 (number of networks) ($\forall i \in \mathcal{I}\ \sum_{j\in\mathcal{J}} w_j^i = 3$). Also, we defined minimum assignable spectrum to network providers $(\tilde{\vartheta}_1, \tilde{\vartheta}_2, \tilde{\vartheta}_3) = (15,25,30)$ while the technology efficiency of the networks $(\eta_1, \eta_2, \eta_3) = (5,8,10)$ and total releasable spectrum for all network providersΛ is 100 ($\Lambda = 100$).

We present three simulations as follow and the aggregate result is presented in Table I.

Simulation 1:

In the first simulation we will divide the available spectrum between all networks equally$(\vartheta_1, \vartheta_2, \vartheta_3) = (100/3\,, 100/3\,, 100/3)$. The result of simulation is depicted inFig.11(a).

Simulation 2:

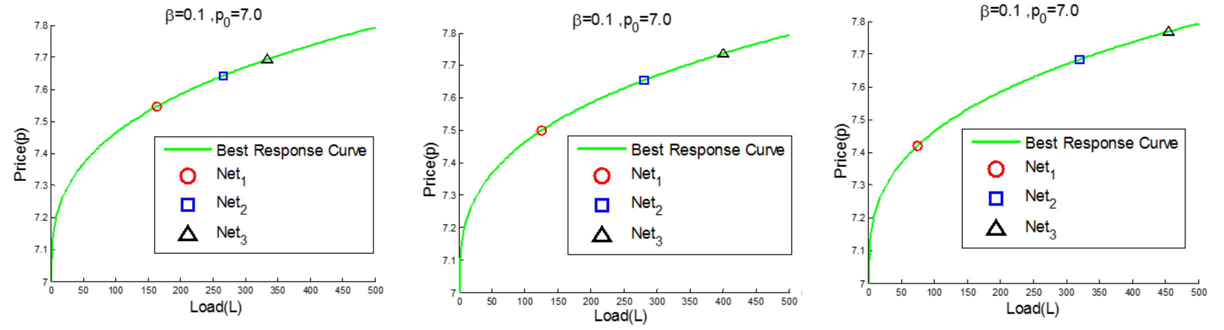
In this simulation we divide the remaining spectrum ($30 = 100 - 15 - 25 - 30$) between all networks equally$(\vartheta_1, \vartheta_2, \vartheta_3) = (25,35,40)$.The result of simulation is depicted inFig.11(b).

Simulation 3:

In this simulation we adjust the available free spectrum (30) using the mechanism in (17). The result of simulation is depicted in Fig.11(c).

Table I shows the result of three different strategies of the regulatory to distribute the available spectrum between three network providers. In simulation 3 when the free spectrum is distributed within different network providers using the proposed strategy, we find that the clients' welfare meets a higher value in Simulation 3 and the QoS is better. As depicted in Table I, all parameters are enhanced using the proposed strategy.

It is telling that the freedom parameter (β) plays an important role in price convergence and the income of regulatory as well as income of network providers. As depicted in Fig.12, while β is increasing the networks are allowed to increase their prices so the price curve of networks increases



a) The converged prices in simulation 1:$(p_1, p_2, p_3) = (7.55, 7.65, 7.70)$ when $(\vartheta_1, \vartheta_2, \vartheta_3) = (33.3, 33.3, 33.3)$

b) The converged prices in simulation 2: $(p_1, p_2, p_3) = (7.55, 7.65, 7.70)$ when $(\vartheta_1, \vartheta_2, \vartheta_3) = (25, 35, 40)$

c) The converged prices in simulation 3:$(p_1, p_2, p_3) = (7.55, 7.65, 7.70)$ when $(\vartheta_1, \vartheta_2, \vartheta_3) = (15.0, 39.5, 45.4)$

Fig.11.The converged prices in simulations

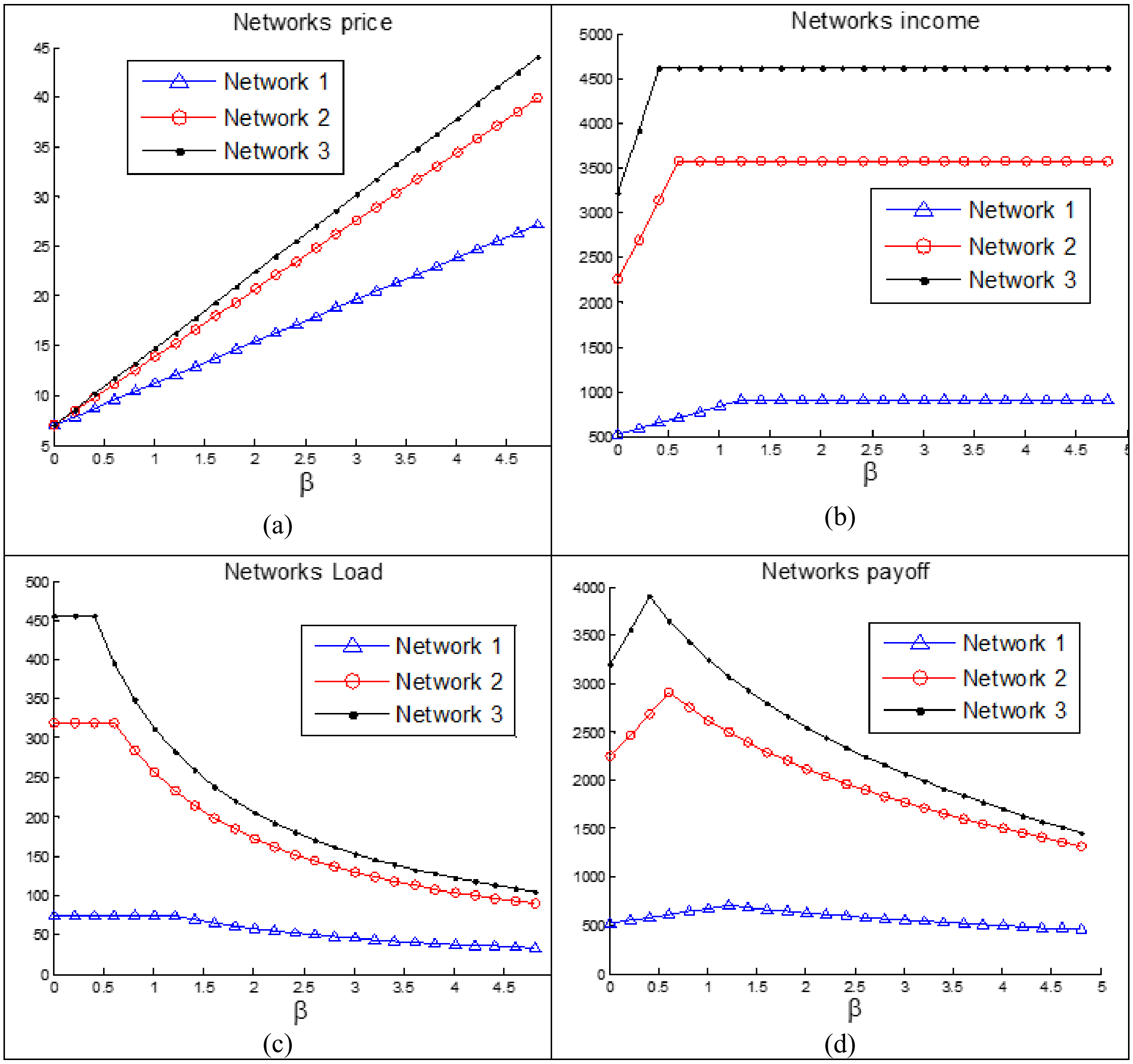
Table I. Results of three simulationswhen = 0.1 , $p_0 = 7.0$

	Simulation 1	Simulation 2	Simulation 3
$(\boldsymbol{\vartheta_1, \vartheta_2, \vartheta_3})$	(33.3 , 33.3 , 33.3)	(25.0 , 35.0 , 40.0)	(15.0 , 39.5 , 45.4)
$(\boldsymbol{p_1, p_2, p_3})$	(7.55,7.65,7.70)	(7.50 , 7.65 , 7.74)	(7.42 , 7.69 , 7.77)
$\boldsymbol{\theta}$	(1.71,1.69,1.67)	(1.62 , 1.59 , 1.57)	(1.68 , 1.49 , 1.47)
Networks Load (L)	(164.8,265.4,332.0)	(124.8,279.8,399.8)	(76.2,317.2,455.2)
Total Delivered Service	762.2	804.3	848.5
Total Clients' payment	9820.1	9776.4	9776.8
Regulatory Income(Tax)	245.5	269.9	299.7
Average of QoS	0.59	0.63	0.67
Welfare*1000	77.62	82.27	86.79

by the parameter β; and according to the clients' limited budget the income of networks increases until clients do not spend all of their budget to buy the service. Therefore, we find that after a special value of β the income of networks is a straight line. As depicted in Fig.12 (c), the load of networks decreases after a special value of β because the clients do not have enough budget to purchase their total required service.

Fig.13 shows the impact of the parameter β on the income of the regulatory. By increase of the value of the parameter β, the income of the regulatory increases because the regulatory receives a higher amount of tax from the network providers.

The impact of the variation of parameter β on the clients' payment and clients' welfare is drawn in Fig.14. Because of the limited value of client budget we find that after a special value of β, clients have to spend the total amount of their budget and so the curve of clients' payment after the increasing phase meets a straight line. The most important result is obtained from Fig.14(b) which shows that the welfare of clients is decreases by the parameter.



(a)

(b)

(c)

(d)

Fig.12. The impact of freedom parameter (β) on a) Networks price, b) Networks income, c) Networks load, d) Networks payoff.

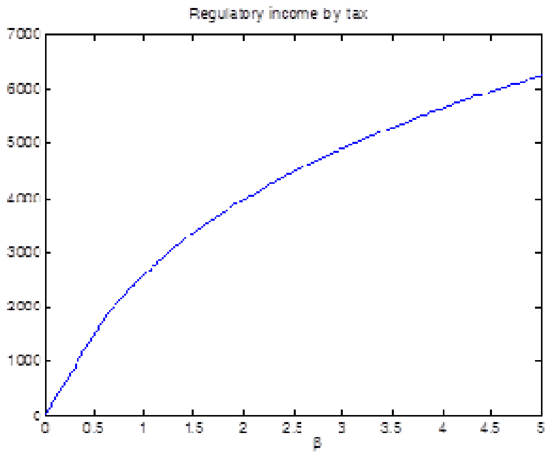
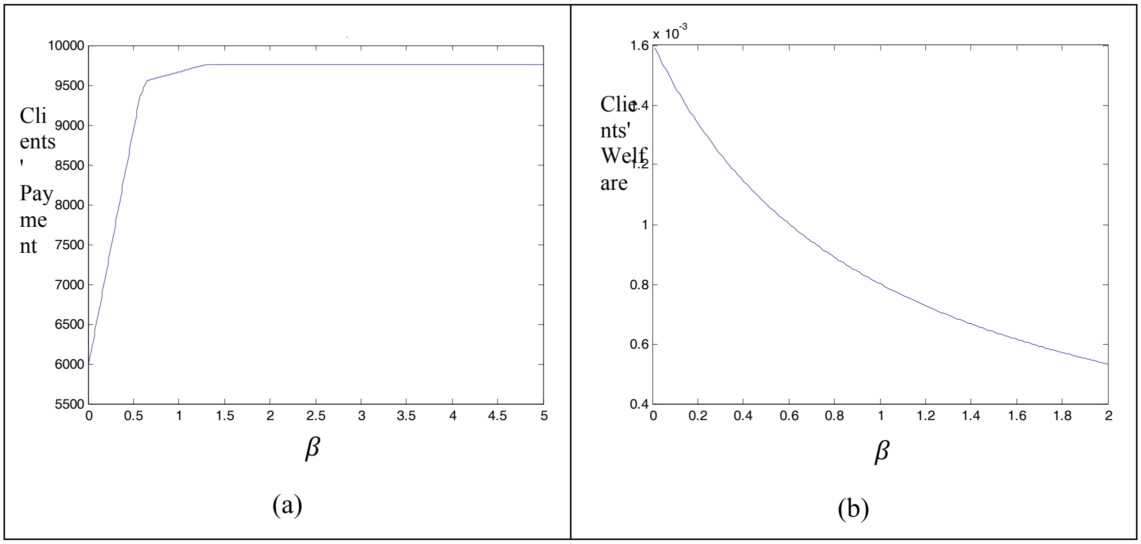


Fig.13. The impact of freedom parameter (β) on the regulatory income.



(a) (b)

Fig.14. The impact of freedom parameter (β) on a) clients' payment and b) clients' welfare.

The regulatory can tune the parameter β to balance its income and clients' welfare.

VI. CONCLUSION

In this paper we present a three stage game to simulate the clients and network provider actions and propose a simple and effective strategy for the regulatory to assign the spectrum to different providers with the goal of increasing the clients' welfare. The regulatory controls the price of service using the taxation mechanism, so, it is not profitable for networks to announce the high price for their service and networks are forced to sell their service as expensive as the regulatory allows.

The experimental results show that by the specified value of freedom parameter (β), the spectrum allocation mechanism results in the best situation that maximizes the clients' welfare. The regulatory is the main player in our proposed game and in future work we will focus on tuning the parameter β

to balance the income of the network and the clients’ welfare. Also in different SAT while we have a limited spectrumamount Γ, the clients' welfare are different and we show, our simple approach for spectrum allocation adjustment leads to increase the clients' welfare.We proved that clients and network providers will follow the proposed mechanism and it is not profitable for anyone to violate the best strategy.

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