# Selective Tone Reservation Method for PAPR Reduction of Spatially Multiplexed OFDM Systems

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Abstract— Tone Reservation (TR) is one of the well-known methods for PAPR reduction of Orthogonal Frequency Division Multiplexing (OFDM) systems. In this method some sub-carriers are reserved for PAPR reduction and the symbols in these sub-carriers are selected such that the PAPR of the OFDM frame is minimized. In this paper, the iterative algorithm introduced earlier for TR method in single antenna OFDM systems is extended to multiple antenna Spatially Multiplexed OFDM (SM-OFDM) systems. In the proposed method, which is called selective TR, in each iteration the peak reduction symbols of the antenna with maximum PAPR are updated. Simulation results show that using this approach leads to the performance improvement in comparison to the case where TR method is applied independently to all of antennas.

Keywords: Spatial Multiplexing (SM), OFDM, PAPR, Tone Reservation (TR).

# 1.INTRODUCTION

OFDM systems are extremely used for broadband wireless communications in frequency selective channels [1]. The effect of broadband frequency selective channels on OFDM signals can be simply equalized while it needs complex equalizers in the case of single carrier signals [1, 2]. However, OFDM signals have high PAPR which leads to the saturation of high power amplifiers. To overcome this problem, several PAPR reduction methods have been proposed. Some of these methods are Selected Mapping (SLM) [3, 4], Partial Transmit Sequences [5, 6], using codes with low PAPR code words [7, 8], clipping and filtering [9-12] and Active Constellation Extension (ACE) [13, 14].

One of the peak reduction methods is Tone Reservation (TR) [15, 16]. In this method, some of the subcarriers of an OFDM frame are reserved for PAPR reduction. The set of the indices of the reserved symbols is called Peak Reduction Tone (PRT) set. Performance of TR method depends on the number of the reserved tones and their locations (PRT set). Finding the optimal PRT set is equivalent to

finding the time domain kernel with the minimum secondary peak. The time domain kernel is obtained by the application of IFFT operation to the signal which is nonzero (with value 1) only in the indices of the PRT set [17, 18]. After selection of optimal PRT set, the symbols transmitted in these subcarriers must be optimized to reduce the PAPR. In [17], an iterative method was proposed for finding the optimum peak reduction symbols. In this method, in each iteration the time domain kernel is shifted to the location of the sample with maximum amplitude and it is added to the original signal with proper phase and amplitude to reduce the peak value.

Using multiple transmitter and receiver antennas, which is named by Multiple Input Multiple Output (MIMO), is a well-known method for the improvement of the Bit Error Rate (BER) and data rate of wireless systems in multipath channels. Using Spatial Diversity (SD) in MIMO systems leads to improvement of BER. Spatial diversity can be achieved using either Space Time Block Codes (STBC) or Space Frequency Block Codes (SFBC). In Spatial Multiplexing (SM) systems several transmitter antennas are used to increase the data rate. In these systems, independent information sequences are transmitted from  $N_t$  transmitter antennas simultaneously. The overall bit rate is  $N_t$ times that of single antenna systems. At the receiver side, the combinations of the transmitted signals are received via several receiver antennas. If the channel matrix is known, then the interference can be mitigated and the transmitted symbols are detected [19]. MIMO –OFDM systems are the combination of MIMO and OFDM techniques. MIMO-OFDM systems also suffer from high PAPR problem. In [20-22] TR method has been used for PAPR reduction of SFBC-OFDM systems. In SM- OFDM systems the combination of SM and OFDM techniques leads to achieving the both advantages of data rate increase and robustness against multipath fading [23]. Several techniques have been proposed to reduce the PAPR of SM-OFDM systems such as directed Selected Mapping (dSLM) [24], using Spatial Shifting (SS) [25] and applying a unitary matrix in frequency domain [26]. In this paper, iterative TR method proposed in [17] is extended to SM-OFDM systems. In this method, in each iteration the antenna with maximum PAPR is selected and its peak reduction symbols are updated such that the peak value is reduced. Simulation results show a performance improvement in PAPR reduction in comparison to the case that the iterative method is applied to the frames of different antennas independently.

The remainder of the paper is organized as follows: Section 2 contains the system model of single antenna OFDM system and a brief introduction of the iterative TR method proposed in [17] for PAPR reduction of single antenna OFDM systems. Then, in section 3 the system model of SM-OFDM systems is discussed. In section 4, the proposed iterative method for PAPR reduction of SM-OFDM systems is introduced. The performance of the proposed method is evaluated by simulation results in section 5.

### 2 .TR METHOD FOR PAPR REDUCTION OF OFDM SIGNALS

In OFDM systems, the input bit stream is mapped to the constellation points. Then, the stream of the complex symbols are separated into sequential frames of the length  $N_c$  symbols. A sample frame is denoted by  $\boldsymbol{X} = [X(0), X(1), \dots, X(N_c - 1)]^T$ . The samples of the time domain OFDM signal,  $\boldsymbol{x} = [x(0), x(1), \dots, x(N - 1)]^T$ , are obtained as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N_c - 1} X(k) e^{j\frac{2\pi kn}{N}} , n = 0, 1, \dots, N - 1$$
(1)

where  $N/N_c$  is the oversampling ratio. The above equation can be written in the matrix form of

 $\mathbf{x} = \mathbf{F}\mathbf{X}$ . The matrix  $\mathbf{F}$  is the IFFT matrix with the elements  $f_{n,k} = \left(\frac{1}{\sqrt{N}}\right)e^{j\frac{2\pi kn}{N}}$ . To avoid interframe interference a Cyclic Prefix (CP) is added at the beginning of the frame  $\mathbf{x}$  as follows:

$$x_{cp}(n) = \begin{cases} x(n) & n = 0, 1, 2, \dots, N - 1\\ x(n+N) & n = -N_G, -N_G + 1, \dots, -1 \end{cases}$$
(2)

 $N_G$  is the length of the CP. The samples  $x_{CP}(n)$  are passed through Analogue to Digital Converter (ADC) with the clock rate of N/T to produce the baseband OFDM signal, x(t),  $-T_G \leq t \leq T$ , where  $T_G$  and T are the time of the CP and the time of an OFDM frame, respectively. The signal x(t) is converted to the passband signal and is transmitted after amplification. The existence of the peaks with high amplitudes in OFDM signal x(t) leads to the saturation of High Power Amplifier (HPA). This phenomena is evaluated by the parameter Peak to Average Power Ratio (PAPR) which is defined by

$$PAPR\{x(t)\} = \frac{\max_{0 \le t \le T}\{|x(t)|^2\}}{E\{|x(t)|^2}$$
(3)

It has been shown in [17] that if the oversampling ratio is greater than 4, then the PAPR of the continuous time signal x(t) can be estimated by the PAPR of the discrete time signal x(n) as follows:

$$PAPR\{\mathbf{x}\} = \frac{\max_{0 \le n \le N-1}\{|x(n)|^2\}}{E\{|x(n)|^2}$$
(4)

In TR method, some of the symbols (Tones), X(k), are reserved for PAPR reduction. If the set  $\Omega$  includes the indices of the Peak Reduction Tones (PRT), then the transmitted symbols are

$$X(k) = \begin{cases} S(k) & k \notin \Omega \\ C(k) & k \in \Omega \end{cases}$$
(5)

Where S(k),  $k \in \Omega$  are data symbols and C(k),  $k \in \Omega$  are the peak reduction symbols. The frequency domain kernel  $\mathbf{P} = [P(0), P(1), \dots, P(N_c - 1)]^T$  is defined by

$$P(k) = \begin{cases} 0 & k \notin \Omega \\ 1 & k \in \Omega \end{cases}$$
(6)

The time domain kernel is defined by  $\mathbf{p} = \mathbf{FP}$ . The maximum value of P(n) is  $p(0) = 1/\sqrt{N} \sum_{k=0}^{N_c-1} P(k) = W/\sqrt{N}$ , where W is the number of peak reduction tones. Two main problems in TR method are the selection of PRT set,  $\Omega$ , and determination of the peak reduction symbols, C(k),  $k \in \Omega$ . In [17] it has been proposed that the PRT set,  $\Omega$ , is selected such that the second peak of P(n) is minimized, i.e.,

$$\Omega_{opt} = \arg\min_{\Omega} |||p(1)|, |p(2)|, \dots, |p(N-1)|||_{\infty}$$
(7)

In [17] an iterative method for determination of the peak reduction symbols has been proposed. In this method, in each iteration the amplitude of one peak of the signal x(n) is reduced. If  $\mathbf{x}^{l}$  and  $\mathbf{c}^{l}$  are the OFDM signals and peak reduction signal after l iterations, respectively then

$$\mathbf{x}^{l} = \mathbf{x}^{l-1} + \mathbf{c}^{l}, \quad \mathbf{x}^{0} = \mathbf{x}.$$
$$\mathbf{c}^{l} = \alpha^{l} \mathbf{p}_{(\tau^{l})} \tag{8}$$

where  $\mathbf{p}_{(\tau^l)}$  is circular shifted version of  $\mathbf{p}$  by  $\tau^l$  units and  $\boldsymbol{\alpha}^l$  is a complex number. The parameters  $\tau^l$  and  $\boldsymbol{\alpha}^l$  is obtained by

$$\tau^{l} = \arg \max_{0 \le n \le N-1} |x^{l-1}(n)|$$

$$|x^{l-1}(\tau^{l}) + \alpha^{l} p(0)| = \zeta$$
(9)

where  $\zeta$  is a threshold level. In fact, in each iteration the kernel p(n) is shifted to the index of the peak value of the signal and it is added to the signal with an opposite phase. It is clear that this algorithm only updates the peak reduction symbols, C(k), and does not change data symbols, S(k), in frequency domain.

## 3. SYSTEM MODEL OF SM-OFDM SYSTEMS AND PAPR PROBLEM

In spatially multiplexed OFDM systems, the input bit stream is interleaved and encoded by a channel encoder, and the encoded bits are mapped onto the constellation points. Then, the symbols are demultiplexed among  $N_t$  transmitter antennas. In each transmission branch, the symbols are grouped into blocks with a length of  $N_c$ , which are the input of the OFDM modulator. If the frame of the *q*th antenna is denoted by Xq,  $q = 1, 2, ..., N_t$ , then the time domain signal of the *q*th antennas is

$$x_q(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N_c - 1} X_q(k) e^{j\frac{2\pi kn}{N}}, \qquad n = 0, 1, \dots, N - 1.$$
(10)

After addition of CP and converting the signal,  $x_q(n)$ , to continuous-time signal,  $x_q(t)$ , it is transmitted from the qth antenna. Using the same bandwidth, data rate of SM-OFDM system is  $N_t$  time of that for single antenna OFDM system. SM-OFDM systems also suffer from high PAPR problem. The PAPR of the SM-OFDM system is defined by the maximum PAPR of the signals of the antennas as follows:

$$PAPR\{\mathbf{x}_{1}, \mathbf{x}_{2}, \dots, \mathbf{x}_{N_{t}}\} = \max_{1 \le q \le N_{t}} PAPR\{\mathbf{x}_{q}\}$$
(11)

Thus the PAPR of SM-OFDM system is larger than that of the single antenna OFDM system.

#### 4 .SELECTIVE TR METHOD FOR PAPR REDUCTION OF SM-OFDM SYSTEMS

Iterative method proposed in [17] can be applied independently to the signals of  $N_t$  antennas,  $\mathbf{x}_q$ ,  $q = 1, 2, ..., N_t$  to reduce the overall PAPR. This approach is called ordinary TR. Since the PAPR of SM-OFDM system is defined by the maximum PAPR of antennas thus the application of TR method to the signals of all antennas will reduce the overall PAPR. The frequency domain signal of the qth antenna is defined by:

$$X_{q}(k) = \begin{cases} S_{q}(k) & k \notin \Omega \\ C_{q}(k) & k \in \Omega \end{cases}$$
(12)

where  $S_q(k)$  and  $C_q(k)$  are data symbols and peak reduction symbols of the qth antenna. It is assumed that the same PRT sets are used in all of antennas. In ordinary TR method, the symbols  $C_q(k)$ ,  $q = 1, 2, ..., N_t$  are determined using M iterations per antenna. Thus the total number of iterations is  $M' = MN_t$ . In this paper, it is proposed that after each iteration the antenna with maximum PAPR is determined and only the peak reduction symbols of that antenna are updated. The steps of the proposed method, which is named by selective TR, can be summarized as follows:

- 1. The time domain signals  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N_t}$  are generated with zero peak reduction symbols  $(C_q(k)=0, k \in \Omega, q = 1, 2, \dots, N_t)$
- 2.  $\mathbf{x}_{q}^{0} = \mathbf{x}_{q}$ ,  $1 \leq q \leq N_{t}$  and l = 1
- 3. The index of the antenna with maximum PAPR is determined by

$$q^{l} = \arg \max_{1 \le q \le N_{t}} \{PAPR\{\mathbf{x}_{q}\}\}$$
(13)

4. The values of  $\tau^{l}$  and  $\alpha^{l}$  are calculated as

$$\tau^{l} = \arg \max_{0 \le n \le N-1} \left| \mathbf{x}_{q}^{l-1}(n) \right|$$

$$\left| \mathbf{x}_{q}^{l-1}(\tau^{l}) + \alpha^{l} \mathbf{p}(0) \right| = \zeta$$
(14)

5. The signal of the  $q^{l}$  th antenna is updated as follows:

$$\mathbf{x}_{ql}^{l} = \mathbf{x}_{ql}^{l-1} + \mathbf{c}^{l}$$
$$\mathbf{c}^{l} = \alpha^{l} \mathbf{p}_{(\tau^{l})}$$
(15)

- 6. Let  $\mathbf{x}_{q}^{l} = \mathbf{x}_{q}^{l-1}$ ,  $q \neq q^{l}$  and l = l + 1.
- 7. If the target overall PAPR is achieved or l > M' stop otherwise go to step 2.

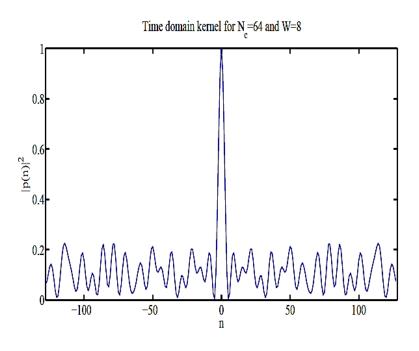


Fig.1. Normalized amplitude of the time domain kernel for TR method with  $N_c = 64$  subcarriers and W = 8 peak reduction tones.

The main difference of the selective TR and ordinary TR methods is that the number of iterations for PAPR reduction of the signal of each antenna is  $M_q = M'/N_t$ ,  $q = 1, 2, ..., N_t$  in ordinary TR method while it is different for different antennas in selective TR method and  $\sum_{q=1}^{N_t} M_q = M'$ .

#### **5**.SIMULATION RESULTS

The performance of the ordinary TR and selective TR methods has been evaluated by simulation. SM-OFDM systems with  $N_t = 2$  and  $N_t = 3$  antennas and  $N_c = 64$  subcarriers are considered. The PRT set is chosen as  $\Omega = \{11, 12, 14, 25, 32, 37, 41, 47\}$  [18]. The normalized amplitude of the time domain kernel, p(n), has been shown in Fig.1. As can be seen in this figure the time domain kernel has low amplitude second peak which is suitable for TR method.

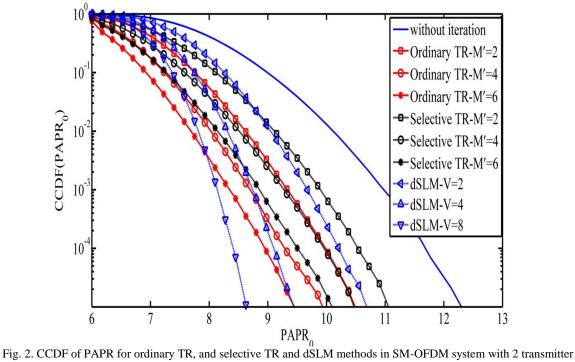
The data symbols are chosen from the 16-QAM constellation points  $\{\pm 1, \pm 3\} + j\{\pm 1, \pm 3\}$ . To evaluate the performance of the PAPR reduction methods, often the Complementary Cumulative Density Function (CCDF) of the PAPR is used which is defined by

$$CCDF(\eta) = Pr\{P \ AP \ R > \eta\}$$
(16)

$N_t = 2$					
Selective TR			dSLM		
M'	СМ	CA	V	СМ	CA
2	512	512	2	1536	3072
4	1024	1024	4	4608	9216
6	1536	1536	8	10752	21504
$N_t = 3$					
3	768	768	2	2304	4608
6	1536	1536	4	6912	13824
9	2304	2304	8	16128	32256

TABLE1. NUMBER OF COMPLEX MULTIPLICATIONS AND ADDITIONS FOR DSLM AND SELECTIVE TR METHODS IN SM-OFDM SYSTEM WITH 64 SUBCARRIERS

where Pr(A) is the probability of the event A. Figures 2 and 3 show the CCDF of the PAPR and the value of power increase using ordinary TR and selective TR methods in an SM-OFDM system with two transmitter antennas. Comparison of the selective TR method with ordinary TR in Fig.2 shows that selective TR method has a performance improvement of 0.6dB, 0.4dB and 0.3dB for the total number of iterations of M' = 2, M' = 4 and M' = 6, respectively. It is obvious that in TR method the extra power transmitted from the peak reduction tones is a penalty that must be paid. Fig.3 shows that the value of the power increase in selective TR method is approximately the same as that for ordinary TR method. In Fig.2 the performance of the proposed method has been compared with dSLM method [24]. In dSLM method, the frequency domain vectors  $X_q, q = 1, 2, ..., N_t$  are multiplied by V different vectors of  $\pm 1$  to generate different representations of information. The representation with minimum PAPR is selected and it is transmitted with side information of the index of the selected vector. In dSLM method  $N_t(V-1)$  additional IFFTs must be performed which needs  $(NN_t(V-1)\log_2 N_c)/2$  Complex Multiplications (CM) and  $NN_t(V-1)\log_2 N_c$  Complex Additions (CA). The proposed selective TR method consists of NM' CMs and NM' CAs. Table. 1 shows the number of required CMs and CAs for dSLM and Selective TR methods. As can be seen from Fig.2 the performances of the proposed method with M' = 6 and dSLM method with V = 4are the same while the complexity of the proposed method is more than 4 times less than dSLM method.



antennas

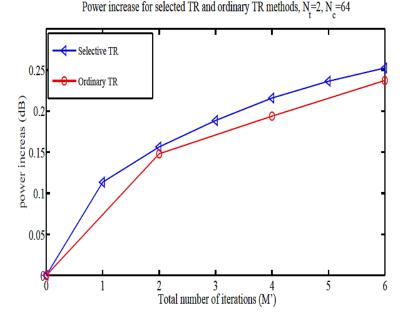


Fig. 3. The values of power increase for ordinary TR and selective TR methods in SM-OFDM system with 2 transmitter antennas versus the total number of iterations.

Figures 4 and 5 show the simulation results for an SM-OFDM system with three transmitter antennas. As can be seen from figure 4, in this case the PAPR reduction performance improvement of selective TR method in comparison to ordinary TR algorithm is about 1dB. Fig.5 shows that the values of the power increase of two methods are approximately the same in SM-OFDM system with

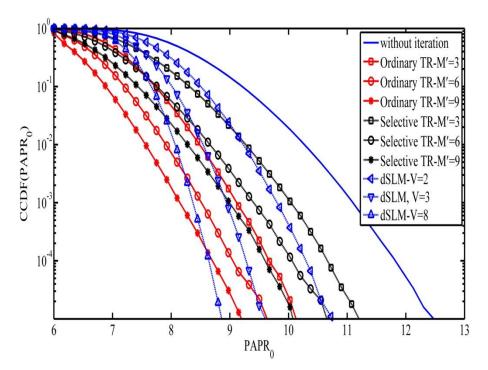


Fig. 4. CCDF of PAPR for ordinary TR, and selective TR and dSLM methods in SM-OFDM system with 3 transmitter antennas

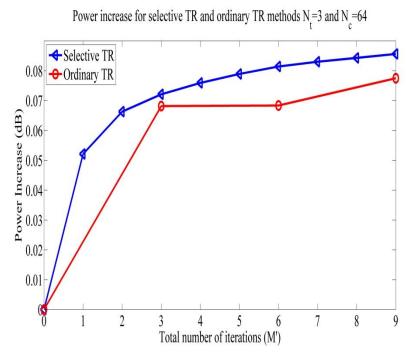


Fig. 5. The values of power increase for ordinary TR and selective TR methods in SM-OFDM system with 3 transmitter antennas versus the total number of iterations.

three transmitter antennas. In this case, the performances of the proposed method with M' = 9 and dSLM method with V = 8 are approximately the same while the complexity of the proposed method is more than 8 times less than dSLM method.

#### **6**.CONCLUSION

In this paper, an algorithm for the calculation of the values of the peak reduction symbols in SM-OFDM systems was proposed. In this algorithm, which was called selective TR method, in each iteration the value of symbols of the antenna with maximum PAPR are only updated. Since the PAPR in SM-OFDM systems is defined as the maximum PAPR of the antennas, this method is expected to have a better performance than that the case the TR method is applied to all of the antennas (This approach was called ordinary TR). Simulation results show that the proposed method has a better performance in PAPR reduction in comparison to the ordinary TR ones. The performance difference of these two methods increases when the number of transmitter antennas is increased. The proposed method can also achieve the performance of dSLM method with a considerable less complexity.

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