# Bounds for Multiple-Access Relay Channels with Feedback Via Two-way Relay Channel

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*Abstract*— **In this study, we introduce a new two-way relay channel and obtain an inner bound and an outer bound for the discrete and memoryless multiple access relay channels with receiver-source feedback via two-way relay channel in which end nodes exchange signals by a relay node. And we extend these results to the Gaussian case. By numerical computing, we show that our inner bound is the same with one obtained by Hou-Koetter-Kramer, for the channel with relay-source feedback.**

*Index Terms*— multiple access relay channel; two-way relay channel; block Markov coding.

## I. INTRODUCTION

Wide using of relay in wireless networks is for its ability to support multiple nodes simultaneously. In many networks such as local area wireless networks/wide area wireless networks, wireless sensor networks, military and monitoring applications where the performance of networks depends on the performance and lifetime of the most connected nodes, the relay node can improve the performance of nodes that transmit data [1].

In [2], multiple access relay channel (MARC) is introduced, where some nodes (sources) communicate with one destination with the help of a relay node (Fig. 1). Gaarder and Wolf [3] have demonstrated that it is possible to increase the capacity region of a discrete memoryless channel through the use of feedback. In [4], an achievable rate for the discrete memoryless relay channel with receiver-transmitter feedback was proposed. In [5], the inner and outer bounds for multiple access relay channels with relay-source feedback were obtained and was showed that feedback in MARC increase the achievable rate region.

In two-way communication two parties send information to each other. The two-way channel was first introduced by Shannon [6], who derived inner and outer bounds on the capacity region of discrete and memoryless channel. Nowadays, there has been an increasing research efforts on two-way relay channel (TWRC) from both academic and industrial communities [7]-[11]. In [12],
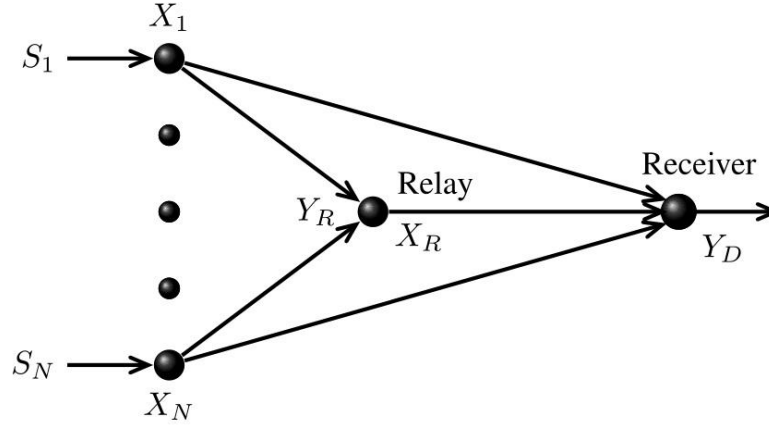
Fig. 1 A N-source multiple access relay channel [5].

achievable rate regions for the TWRC in different strategies were obtained in which two nodes communicate simultaneously in both directions with the help of one relay.

## II.   CHANNEL DEFINITIONS AND MAIN RESULTS

### A.   *Two-way (in the sense of shannon) relay channel*

We consider a two way relay channel with four nodes as Fig. 2 and suppose all nodes are full-duplex. Node $T_1$ and $T_2$ want to exchange messages with node $T_4$ in direct path via node $T_3$ as a relay that has no own messages to transmit.

$W_1 \in \mathcal{W}_1$, $W_2 \in \mathcal{W}_2$ and $W_4 \in \mathcal{W}_4$ are transmitted messages from terminal $T_1$, terminal $T_2$ and terminal $T_4$, respectively. $X_{i,t} \in \mathcal{X}$ and $Y_{i,t} \in \mathcal{Y}$, $i \in \{1,2,3,4\}$, $t \in \{1,2,\cdots,n\}$, are channel inputs and outputs, respectively. We assume a time invariant and memoryless two-way relay channel which is defined by conditional channel distribution $P(y_1, y_2, y_3, y_4 \mid x_1, x_2, x_3, x_4)$, where $X_i$ and $Y_i$ $(i = 1,2,3,4)$ are random variables representing the respective channel inputs and outputs.

In our work, transmitted symbols of terminals $T_1$ and $T_2$ are functions of their messages and their past channel outputs but transmit symbol of terminal $T_4$ is a function of its channel output where in our work, it is an identity function. The transmitted symbol from terminal $T_3$ is a function of its past channel output. In forward direction, terminals $T_1$ and $T_2$ send signals to terminal $T_4$ via terminal $T_3$, and in backward direction $T_4$ sends signal to terminals $T_1$ and $T_2$ via terminal $T_3$ simultaneously.

$$X_{1,t} = f_{1,t}(W_1, Y_1^{t-1}) \tag{1}$$
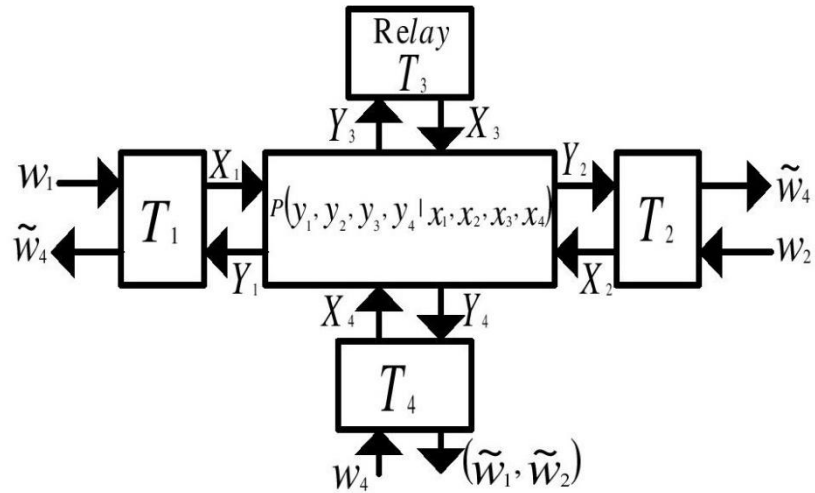$$X_{2,t} = f_{2,t}(W_2, Y_2^{t-1}) \tag{2}$$

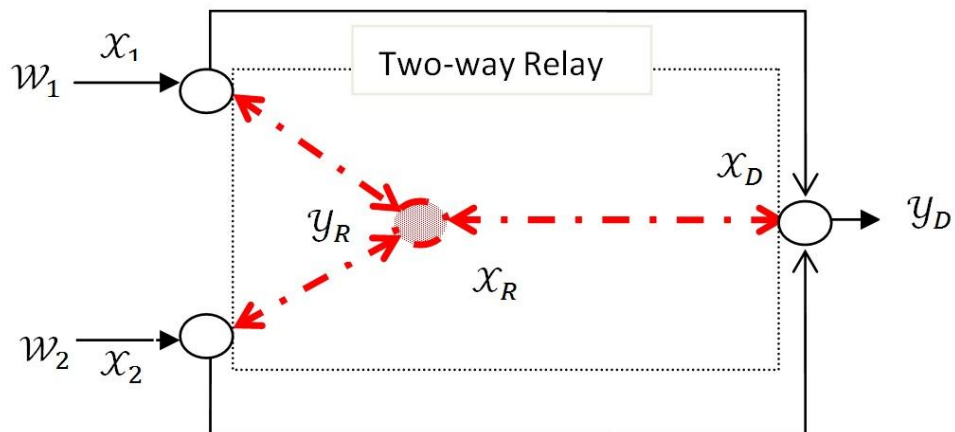Fig. 2. Two-way relay channel with four nodes.



Fig. 3.  2-source multiple-access relay channel with feedback between receiver-source via two-way relay channel.

$$X_{3,t} = f_{3,t}(Y_3^{t-1}) \tag{3}$$

$$X_{4,t} = f_{4,t}(Y_{4,t}) = f'_{4,t}(W_4) \tag{4}$$

### III.  MARC WITH FEEDBACK

We study the two-source MARC with receiver-source feedback via two-way relay channel as shown in Fig. 3. A $(2^{nR_1}, 2^{nR_2}), n)$ code, for two-source MARC with feedback from the receiver consists of two sets of integers $w_1 \in [1:2^{nR_1}]$ and $w_2 \in [1:2^{nR_2}]$, called the message sets; three encoding functions (according to (1),(2) and (4)),

$$f_1 : (\mathcal{W}_1, \mathcal{Y}_D^n) \to \mathcal{X}_1^n \tag{5}$$

$$f_2 : (\mathcal{W}_2, \mathcal{Y}_D^n) \to \mathcal{X}_2^n \tag{6}$$

$$f_D : \mathcal{Y}_D^n \to \mathcal{X}_D^n \tag{7}$$

a set of relay functions $\{f_t\}_{t=1}^n$ such that (according to (3))

$$X_{R,t} = f_t\{Y_R^{t-1}\}, \qquad 1 \le t \le n, \tag{8}$$

and a decoding function,

$$g : \mathcal{Y}_D^n \to \mathcal{W}_1 \times \mathcal{W}_2 \tag{9}$$

The index $w_1$ is chosen uniformly distributed over $[1 : 2^{nR_1}]$ and corresponding codeword is sent over the channel by sender 1. Another sender does likewise. Relay helps the receiver with decoding and forwarding of sending information. We define the average probability of error for the $((2^{nR_1}, 2^{nR_2}), n)$ code as follows

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \times \sum_{(w_1,w_2) \in \mathcal{W}_1 \times \mathcal{W}_2} Pr[g(Y_D^n) \ne (w_1, w_2) \,|\, (w_1, w_2) \, has.been.sent] \tag{10}$$

A rate $(R_1, R_2)$ is said to be achievable for the multiple-access relay channel with feedback from receiver via two-way relay channel if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ code with $P_e^{(n)} \to 0$.

*a)      Previous Related Work Results:*

**Proposition 1** The achievable rate region for the MARC with relay-source feedback, established in [5], is given by:

$$\bigcup (R_1, R_2) \in \mathbb{R}_+^2$$
$$R_1 \le I(X_1; Y_R \,|\, X_2, X_R, U)$$
$$R_2 \le I(X_2; Y_R \,|\, X_1, X_R, U)$$
$$R_1 + R_2 \le \min\{I(X_1, X_2; Y_R, | X_R, U), I(X_1, X_2, X_R; Y_D)\}$$

where the union is taken over
$$p(u, x_1, x_2, x_R, y_R, y_D) = p(u)p(x_1 \,|\, u)p(x_2 \,|\, u)p(x_R \,|\, u)$$
$$\times p(y_R, y_D \,|\, x_R, x_1, x_2) \tag{11}$$

**Proposition 2** The achievable rate region for the Gaussian MARC with relay-source feedback, established in [5], is given by:

$$\bigcup_{\substack{0 \le \alpha_1 \le 1 \\ 0 \le \alpha_2 \le 1}} \begin{cases} (R_1, R_2) \in \mathbb{R}_+^2 \\ R_1 \le C(\dfrac{\alpha_1 P_1}{N_1})1mm \\ R_2 \le C(\dfrac{\alpha_2 P_2}{N_1})1mm \\ R_1 + R_2 \le \min \{ C(\dfrac{\alpha_1 P_1 + \alpha_2 P_2}{N_1}), \\ C(\dfrac{\alpha_1 P_1 + \alpha_2 P_2 + (\sqrt{\overline{\alpha_1} P_1} + \sqrt{\overline{\alpha_2} P_2} + \sqrt{P_R})^2}{N_2}) \} \end{cases}$$

where $C(x) = 0.5 \log_2(1+x)$.

## IV. MAIN THEOREMS

***Theorem 1:*** The capacity region $\mathcal{C}_{FB-MARC}$ of MARC with feedback from receiver via two-way relay channel is contained in the following set,

$$\bigcup \{(R_1, R_2):$$
$$R_1 \le \min \{ I(X_1; Y_R, Y_D \mid X_D, X_2, X_R),$$
$$I(X_1, X_R; Y_D \mid X_D, X_2) \} \tag{12}$$
$$R_2 \le \min \{ I(X_2; Y_R, Y_D \mid X_D, X_1, X_R),$$
$$I(X_2, X_R; Y_D \mid X_D, X_1) \} \tag{13}$$
$$R_1 + R_2 \le \min \{ I(X_1, X_2; Y_R, Y_D \mid X_D, X_R),$$
$$I(X_1, X_2, X_R; Y_D \mid X_D) \} \} \tag{14}$$

where the union is taken over $p(y_D, y_R, x_1, x_2, x_R, x_D)$.

***Theorem 2:*** An achievable rate region $\mathcal{R}_{FB-MARC}$ for the MARC with feedback from receiver via two-way relay channel is given by following set,

$$\bigcup \{(R_1, R_2):$$
$$R_1 \le \min \{ I(X_1; Y_R \mid V, X_D, X_2, X_R),$$
$$I(X_1; Y_D \mid V, X_R, X_D, X_2) \} \tag{15}$$
$$R_2 \le \min \{ I(X_2; Y_R \mid V, X_D, X_1, X_R),$$
$$I(X_2; Y_D \mid V, X_R, X_D, X_1) \} \tag{16}$$
$$R_1 + R_2 \le \min \{ I(X_1, X_2; Y_R \mid V, X_D, X_R),$$
$$I(X_1, X_2, X_R, X_D; Y_D) \} \} \tag{17}$$

where the union is taken over the all joint distribution of the following form,
$$p(v, y_D, y_R, x_1, x_2, x_R, x_D) = p(v)p(x_1 \mid v, x_R)p(x_2 \mid v, x_R)$$
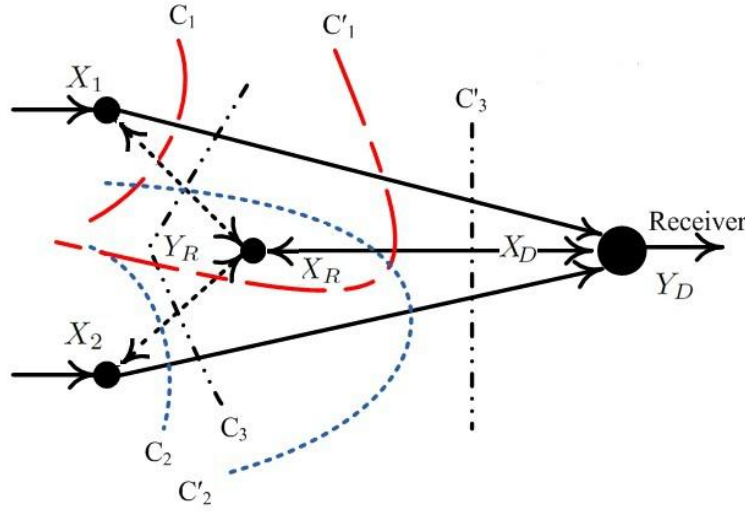$$\times p(x_R \mid v)p(x_D \mid v, x_1, x_2, x_R)p(y_D, y_R \mid x_1, x_2, x_R, x_D) \tag{18}$$

Fig. 4. Illustration of cut sets for MARC [1].

V.  PROOF OF MAIN RESULTS

*A.  Proof of Theorem 1:*

      Theorem 1 is a special case of the cut-set bound [13] (Fig. 4). The details of proof are omitted for the brevity.

*B.  Proof of Theorem 2:*

We consider B blocks, each of n symbols. We use superposition block Markov coding. A sequence of B messages $w_{1,i} \times w_{2,i}$, $i \in \{1,2,\cdots,B\}$ will be sent over the channel in nB transmissions. In each n-block, $b = 1,2,\cdots,B+2$, we use the same set of codebooks:

$$C = \{x_R^n(u,l), x_1^n(u,l,k), x_2^n(u,l,m), v^n(u), x_D^n(u,l,k,m)\}$$

$$k \in [1:2^{nR_1}], m \in [1:2^{nR_2}] l \in [1:2^{nR_1}] \times [1:2^{nR_2}], u \in [1:2^{nR_1}] \times [1:2^{nR_2}] \qquad (19)$$

Random         codebook         generation:         First         fix         a         choice         of $p(v)p(x_1 | v, x_R)p(x_2 | v, x_R)p(x_R | v)p(x_D | v, x_1, x_2, x_R)$.

  1. Generate $2^{n(R_1+R_2)}$ independent identically distributed $n$-sequences $v^n$, each drawn according to $p(v^n) = \prod_{t=1}^{n} p(v_t)$ . Index them as $v^n(u), u = (u_1, u_2) \in [1:2^{nR_1}] \times [1:2^{nR_2}]$.

  2. For  each  $v^n(u)$,  generate  $2^{n(R_1+R_2)}$  conditionally  independent  $n$-sequences  $x_R^n$  with $p(x_R^n | v^n(u)) = \prod_{t=1}^{n} p(x_{R,t} | v_t(u))$. Index them as $x_R^n(u,l), l = (l_1, l_2) \in [1:2^{nR_1}] \times [1:2^{nR_2}]$.

3. For each $(x_R^n(u,l), v^n(u))$, generate $2^{nR_1}$ conditionally independent $n$-sequences $x_1^n$ drawn according to $P(x_1^n \mid x_R^n(u,l), v^n(u)) = \prod_{t=1}^{n} p(x_{1,t} \mid x_{R,t}(u,l), v_t(u))$. Index them as $x_1^n(u,l,k), k \in [1 : 2^{nR_1}]$.

4. For each $(x_R^n(u,l), v^n(u))$, generate $2^{nR_2}$ conditionally independent $n$-sequences $x_2^n$ drawn according to $P(x_2^n \mid x_R^n(u,l), v^n(u)) = \prod_{t=1}^{n} p(x_{2,t} \mid x_{R,t}(u,l), v_t(u))$. Index them as $x_2^n(u,l,m), m \in [1 : 2^{nR_2}]$.

5. For each $(x_1^n(u,l,k), x_2^n(u,l,m))$, choose an $x_D^n$ drawn according to $P(x_D^n \mid x_1^n(u,l,k), x_2^n(u,l,m)) = \prod_{t=1}^{n} p(x_{D,t} \mid x_{1,t}(u,l,k), x_{2,t}(u,l,m))$. Index them as $x_D^n(u,l,k,m)$.

*Encoding, Decoding and Error Analaysis:* Encoding is performed in $B+2$ blocks, The coding strategy is shown in Table I. In Table I, $u_b = (u_{1,b-2}, u_{2,b-2}) = (w_{1,b-2}, w_{2,b-2})$ and $l_b = (l_{1,b-1}, l_{2,b-1}) = (w_{1,b-1}, w_{2,b-1})$.

*Source Terminals:*
The messages are split into B equally sized blocks $w_{1,b}, w_{2,b}, b = 1,2,\cdots,B$. In block $b = 1,2,\cdots,B+2$, the sender $i$ , $i = 1,2$, transmits $x_{i,b}^n(u_b, l_b, w_{1,b}) = \sqrt{\theta_i}\, x_{Rf,i,b}^n(l_b) + \sqrt{\varphi_i}\, x_{i,b}^n(u_b, w_{i,b})$ where $x_{Rf}^n$ denotes the relay's codeword associated to the forward direction. $\theta_i$ and $\varphi_i$ are scaling coefficients and $w_{i,-1} = w_{i,0} = w_{i,B+1} = w_{i,B+2} = 1$. And the receiver send $x_{D,b}^n(u_b, l_b, w_{1,b}, w_{2,b})$ in backward direction.

Let $Y_{D,b-1}^n$ be the observed symbol at sources in block b, source 1 tries to find $\widetilde{w}_{2,b} = w_{2,b}$ in block b such that'

$$(x_{1,b}^n(u_b, l_b, w_{1,b}), x_{2,b}^n(u_b, l_b, \widetilde{w}_{2,b}), v_b^n(u_b), x_{R,b}^n(u_b, l_b),$$

$$x_{D,b}^n(u_b, l_b), y_{D,b-1}^n) \in A_\varepsilon^n(X_1, X_2, V, X_R, X_D, Y_D) \qquad (20)$$

is satisfied.

It can be shown that the probability of error is arbitrarily small if

$$R_2 \leq I(X_2; Y_D \mid V, X_1, X_D, X_R) \qquad (21)$$

Also, source 2 tries to find $\widetilde{w}_{1,b} = w_{1,b}$ in block b with arbitrarily small probability of error if

$$R_1 \leq I(X_1; Y_D \mid V, X_2, X_D, X_R) \qquad (22)$$

Table I. Encoding Strategy

| Block 1 | Block 2 | ... | Block B+1 | Block B+2 |
|---|---|---|---|---|
| $v_1^n(u_1)$ | $v_2^n(u_2)$ | ... | $v_{B+1}^n(u_{B+1})$ | $v_{B+2}^n(u_{B+2})$ |
| $x_{R,1}^n(u_1,l_1)$ | $x_{R,2}^n(u_2,l_2)$ | ... | $x_{R,B+1}^n(u_{B+1},l_{B+1})$ | $x_{R,B+2}^n(u_{B+2},1)$ |
| $x_{1,1}^n(u_1,l_1,w_{1,1})$ | $x_{1,2}^n(u_2,l_2,w_{1,2})$ | ... | $x_{1,B+1}^n(u_{B+1},l_{B+1},1)$ | $x_{1,B+2}^n(u_{B+2},1,1)$ |
| $x_{2,1}^n(u_1,l_1,w_{2,1})$ | $x_{2,2}^n(u_2,l_2,w_{2,2})$ | ... | $x_{2,B+1}^n(u_{B+1},l_{B+1},1)$ | $x_{2,B+2}^n(u_{B+2},1,1)$ |
| $x_{D,1}^n(u_1,l_1)$ | $x_{D,2}^n(u_2,l_2)$ | ... | $x_{D,B+1}^n(u_{B+1},l_{B+1})$ | $x_{D,B+2}^n(u_{B+2},1)$ |

*Relay Terminal:*

*Encoding:* After the transmission of block b is completed, the relay can estimate $(\widetilde{w}_{1,b},\widetilde{w}_{2,b})$ and send

$$x_{R,b+1}^n(u_{b+1},l_{b+1}) = \sqrt{\phi_1}\, x_{Rf,1,b+1}^n(l_{b+1}) + \sqrt{\phi_2}\, x_{Rf,2,b+1}^n(l_{b+1}) + \sqrt{\phi_D}\, x_{Rb,b+1}^n(u_{b+1})$$ in block b+1, where $x_{Rb}^n$

denotes the relay's codeword associated to the backward direction and is equal to $x_{Db}^n$. $\phi_i$, $i=1,2$, and

$\phi_D$ are scaling coefficients

*Decoding:* After the transmission of block b is completed, the relay has seen $y_{R,b}^n$. The relay tries to

find $(\widetilde{w}_{1,b},\widetilde{w}_{2,b})$ such that,

$$(x_{1,b}^n(u_b,l_b,\widetilde{w}_{1,b}), x_{2,b}^n(u_b,l_b,\widetilde{w}_{2,b}), v_b^n(u_b), x_{R,b}^n(u_b,l_b),$$

$$x_{D,b}^n(u_b,l_b), y_{R,b}^n) \in A_\varepsilon^n(X_1,X_2,V,X_R,X_D,Y_R) \tag{23}$$

It can be shown that the relay can decode reliably if,

$$R_1 \le I(X_1;Y_R \mid V,X_2,X_R,X_D) \tag{24}$$

$$R_1 \le I(X_2;Y_R \mid V,X_1,X_R,X_D) \tag{25}$$

$$R_1 + R_2 \le I(X_1,X_2;Y_R \mid V,X_R,X_D) \tag{26}$$

*Sink Terminal:*

The receiver can decode in backward decode technique. In block $B+2$, the receiver declares

$\widetilde{u}_{B+2} = u_{B+2}$ if $\widetilde{u}_{B+2} = (\widetilde{u}_{1,B},\widetilde{u}_{2,B}) = (\widetilde{w}_{1,B},\widetilde{w}_{2,B})$ is the unique one that satisfies the typicality checks

in block $B+2$

$$(x_{1,b}^n(\widetilde{u}_{B+2},1,1),\, x_{2,b}^n(\widetilde{u}_{B+2},1,1),\, x_{R,b}^n(\widetilde{u}_{B+2},1),$$

$$x_{D,b}^n(\widetilde{u}_{B+2},1),\, y_{D,B+2}^n) \in A_\varepsilon^n(X_1,X_2,X_R,X_D,Y_D) \tag{27}$$

In block $B+1$, the receiver already has the estimates for $u_{B+2}=l_{B+1}=(l_{1,B},l_{2,B})=(w_{1,B},w_{2,B})$, it looks for the unique one $\widetilde{u}_{B+1}=(\widetilde{u}_{1,B-1},\widetilde{u}_{2,B-1})=(\widetilde{w}_{1,B-1},\widetilde{w}_{2,B-1})$ that in block $B+1$ satisfies the typicality checks in block $B+1$

$$(x_{1,b}^n(\widetilde{u}_{B+1},l_{B+1},1),\, x_{2,b}^n(\widetilde{u}_{B+1},l_{B+1},1),\, x_{R,b}^n(\widetilde{u}_{B+1},l_{B+1}),$$

$$x_{D,b}^n(\widetilde{u}_{B+1},l_{B+1}),\, y_{D,B+1}^n) \in A_\varepsilon^n(X_1,X_2,X_R,X_D,Y_D) \tag{28}$$

In block $B$, the receiver already has the estimates for $u_{B+1}=l_B=(l_{1,B-1},l_{2,B-1})=(w_{1,B-1},w_{2,B-1})$, it looks for the unique one $\widetilde{u}_B=(\widetilde{u}_{1,B},\widetilde{u}_{2,B})=(\widetilde{w}_{1,B},\widetilde{w}_{2,B})$ that in block $B$ satisfies the typicality checks in block $B$

$$(x_{1,b}^n(\widetilde{u}_B,l_B,w_{1,B}),\, x_{2,b}^n(\widetilde{u}_{B+1},l_{B+1},w_{2,B}),\, x_{R,b}^n(\widetilde{u}_{B+1},l_{B+1}),$$

$$x_{D,b}^n(\widetilde{u}_{B+1},l_{B+1}),\, y_{D,B}^n) \in A_\varepsilon^n(X_1,X_2,X_R,X_D,Y_D) \tag{29}$$

Continuing in this way, The receiver declares all $u_b, 1<b<B+2$, with arbitrarily small probability of error if,

$$R_1+R_2 \le I(X_1,X_2,X_R,X_D;Y_D) \tag{30}$$

Therefore, from (21)-(30) the achievability is proved.

## VI. THE GAUSSIAN CASE

In this section, we extend the obtained results in the previous section to the Gaussian case. The received symbols at the relay and receiver nodes are defined by

$$Y_R = X_1 + X_2 + X_D + Z_1 \tag{31}$$

$$Y_D = X_1 + X_2 + X_R + Z_2 \tag{32}$$

We assume that all channel's gains are unit. The $Z_1$ and $Z_2$ are independent additive white Gaussian noises with zero mean and variance $N_1$ and $N_2$ at the relay and receiver, respectively. The channel input sequences are subject to the following average power constraints

$$\frac{1}{n}\sum_{t=1}^n E[X_{D,t}^2] \le P_D \tag{33}$$

$$\frac{1}{n}\sum_{t=1}^n E[X_{R,t}^2] \le P_R \tag{34}$$

$$\frac{1}{n}\sum_{t=1}^{n}E[X_{i,t}^{2}]\leq P_{i}, \quad i=1,2 \tag{35}$$

We define suitably the Gaussian inputs and auxiliary random variables. Let

$X_{R}\sim\mathcal{N}(0,P_{R}),X_{1}\sim\mathcal{N}(0,P_{1}),X_{2}\sim\mathcal{N}(0,P_{2}),X_{D}\sim\mathcal{N}(0,P_{D}),X_{Rf,1}\sim\mathcal{N}(0,\alpha_{1}P_{1}),$

$X_{Rf,2}\sim\mathcal{N}(0,\alpha_{2}P_{2}),\hat{X}_{1}\sim\mathcal{N}(0,\overline{\alpha}_{1}P_{1}),\hat{X}_{2}\sim\mathcal{N}(0,\overline{\alpha}_{1}P_{2}),$ and $V\sim\mathcal{N}(0,1)$ . Where $\overline{\alpha}_{i}=1-\alpha_{i}$,

$i=1,2,$ . We generate random variables $V,X_{R},X_{1}$ and $X_{2}$ according (18) and as following way:

$$V=\sqrt{\frac{\gamma}{\alpha_{1}P_{1}}}X_{Rf,1}+\sqrt{\frac{\overline{\gamma}}{\alpha_{2}P_{2}}}X_{Rf,2} \tag{36}$$

$$X_{R}=\sqrt{\beta P_{R}}V+\sqrt{\frac{\overline{\beta}P_{R}}{P_{D}}}X_{D}, \tag{37}$$

$$X_{i}=X_{Rf,i}+\hat{X}_{i} \quad ,i=1,2 \tag{38}$$

where $\overline{\beta}=1-\beta$ and $\overline{\gamma}=1-\gamma$ .

***Theorem 3:*** An achievable rate region $\mathcal{R}_{FB-GMARC}$ for the Gaussian MARC with feedback from receiver via two-way relay channel is given by following set,

$$\bigcup(R_{1},R_{2}):$$

$$R_{1}\leq\min\left(C(\frac{(1-\alpha_{1}\gamma)P_{1}}{N_{1}}),C(\frac{(1-\alpha_{1}\gamma)P_{1}}{N_{2}})\right) \tag{39}$$

$$R_{2}\leq\min\left(C(\frac{(1-\alpha_{2}\overline{\gamma})P_{2}}{N_{1}}),C(\frac{(1-\alpha_{2}\overline{\gamma})P_{2}}{N_{2}})\right) \tag{40}$$

$$R_{1}+R_{2}\leq\min\left(C(\frac{P_{1}+P_{2}+P_{R}+2\rho_{1}\sqrt{P_{1}P_{R}}+2\rho_{2}\sqrt{P_{2}P_{R}}}{N_{2}}),\right.$$

$$\left.C(\frac{(1-\alpha_{1}\gamma)P_{1}+(1-\alpha_{2}\overline{\gamma})P_{2}-2\sqrt{\alpha_{1}\alpha_{2}\gamma\overline{\gamma}P_{1}P_{2}}}{N_{1}})\right) \tag{41}$$

where the union is taken over the all joint distribution of the form (18). And, $0\leq\rho_{1}=\sqrt{\alpha_{1}\beta\gamma}\leq1$ and

$0\leq\rho_{2}=\sqrt{\alpha_{2}\beta\overline{\gamma}}\leq1$ , are the correlation coefficients between $x_{1}^{n}$ and $x_{2}^{n}$ with $x_{R}^{n}$ , respectively.

***Theorem 4:*** The capacity region $\mathcal{C}_{FB-GMARC}$ of Gaussian MARC with feedback from receiver via two-way relay channel is contained in the following set,

$$\bigcup(R_{1},R_{2}):$$

$$R_{1}\leq\min\left(C(\frac{P_{1}+P_{R}+2\rho_{1}\sqrt{P_{1}P_{R}}}{N_{2}}),\right.$$

$$C(P_1(1-\rho_1^2)(\frac{1}{N_1}+\frac{1}{N_2})) \tag{42}$$

$$R_2 \leq \min(C(\frac{P_2+P_R+2\rho_2\sqrt{P_2P_R}}{N_2}),$$

$$C(P_2(1-\rho_2^2)(\frac{1}{N_1}+\frac{1}{N_2})) \tag{43}$$

$$R_1+R_2 \leq \min(C(\frac{P_1+P_2+P_R+2\rho_1\sqrt{P_1P_R}+2\rho_2\sqrt{P_2P_R}}{N_2}),$$

$$C((P_1+P_2-\rho_1^2P_1-\rho_2^2P_2)(\frac{1}{N_1}+\frac{1}{N_2})) \tag{44}$$

where the union is taken over $p(y_D, y_R, x_1, x_2, x_R, x_D)$.

### a)    *Proof of Theorem 3:*

For the brevity, we prove only one of important terms.

$$R_1+R_2 \leq I(X_1,X_2;Y_R \mid V,X_D,X_R) \tag{45}$$

$$= h(Y_R \mid V,X_D,X_R) - h(Y_R \mid X_1,X_2,V,X_D,X_R) \tag{46}$$

$$= h(X_1+X_2+X_D+Z_1 \mid V,X_D,X_R) - h(Z_1) \tag{47}$$

$$= h(X_1+X_2+Z_1 \mid V,X_D,X_R) - h(Z_1) \tag{48}$$

$$\overset{(a)}{=} h(X_1+X_2+Z_1 \mid V,X_R) - h(Z_1) \tag{49}$$

where $(a)$ follows from the independency of $X_1+X_2+Z_1$ from $X_D$ when $V,X_R$ are given (Markovity of $(X_1,X_2) \to (V,X_R) \to X_D$).

Let $\mathbb{M}$ be the covariance matrix of $A = X_1+X_2+Z_1$, $V$ and $X_R$ as follows.

$$\mathbb{M} \overset{\Delta}{=} \begin{bmatrix} \overline{\mathbb{E}[A^2]} & \mathbb{E}[AX_R] & \mathbb{E}[AV] \\ \mathbb{E}[X_RA] & \overline{\mathbb{E}[X_R^2]} & \overline{\mathbb{E}[X_RV]} \\ \mathbb{E}[VA] & \overline{\mathbb{E}[VX_R]} & \overline{\mathbb{E}[V^2]} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{21}^T \\ m_{21} & \mathbb{M}_{22} \end{bmatrix} \tag{50}$$

such that sub-matrix $\mathbb{M}_{22}$ is a two dimensional matrix and $m_{21}$ is a row vector. Now, $R_1+R_2$ is bounded as follows.

$$R_1 + R_2 \leq h(A \mid X_R, V) - h(Z_1) \tag{51}$$

$$\leq \frac{1}{2} \log(2\pi e)^2 \det(cov(A \mid X_R, V)) - \frac{1}{2} \log 2\pi e N_1 \tag{52}$$

$$\leq \frac{1}{2} \log\left((2\pi e)^2 \det(m_{11} - m_{21}^T \mathbb{M}_{22}^T m_{21})\right) - \frac{1}{2} \log 2\pi e N_1 \tag{53}$$

$$= \frac{1}{2} \log\left(1 + \frac{(1-\alpha_1\gamma)P_1 + (1-\alpha_2\bar{\gamma})P_2 - 2\sqrt{\alpha_1\alpha_2\gamma\bar{\gamma}P_1P_2}}{N_1}\right) \tag{54}$$

b)     *Proof of Theorem 4:*

For the brevity, we prove only one of important terms.

$$R_1 \leq I(X_1; Y_D, Y_R \mid X_2, X_R, X_D) \tag{55}$$

$$= h(Y_R, Y_D \mid X_R, X_2) - h(Y_R, Y_D \mid X_R, X_2, X_D) \tag{56}$$

$$= h(X_1 + X_2 + X_D + Z_1, X_1 + X_2 + X_R + Z_2 \mid X_2, X_R, X_D) - h(Z_1, Z_2) \tag{57}$$

$$= h(X_1 + Z_1, X_1 + Z_2 \mid X_2, X_R, X_D) - h(Z_1, Z_2) \tag{58}$$

$$\overset{(a)}{\leq} h(X_1 + Z_1, X_1 + Z_2 \mid X_R) - h(Z_1, Z_2) \tag{59}$$

where $(a)$ follows from the removing conditioning.

Let $\mathbb{M}$ be the covariance matrix of $A = X_1 + Z_1$, $B = X_1 + Z_2$ and $X_R$ as follows,

$$\mathbb{M} \overset{\Delta}{=} \begin{bmatrix} \begin{array}{cc|c} \mathbb{E}[A^2] & \mathbb{E}[AB] & \mathbb{E}[AX_R] \\ \mathbb{E}[BA] & \mathbb{E}[B^2] & \mathbb{E}[BX_R] \\ \hline \mathbb{E}[X_R A] & \mathbb{E}[X_R B] & \mathbb{E}[X_R^2] \end{array} \end{bmatrix} = \begin{bmatrix} \mathbb{M}_{11} & m_{21}^T \\ m_{21} & m_{22} \end{bmatrix} \tag{60}$$

such that sub-matrix $\mathbb{M}_{11}$ is a two dimensional matrix and $m_{21}$ is a row vector. Now, $R_1$ is bounded as follows.

$$R_1 \leq h(A, B \mid X_R) - h(Z_1, Z_2) \tag{61}$$

$$\leq \frac{1}{2} \log(2\pi e)^2 \det(cov(A, B \mid X_R)) - \frac{1}{2} \log(2\pi e)^2 \det(cov(Z_1, Z_2)) \tag{62}$$

$$\leq \frac{1}{2} \log\left((2\pi e)^2 \det(\mathbb{M}_{11} - m_{21}^T m_{22}^T m_{21})\right) - \frac{1}{2} \log(2\pi e)^2 (N_1 N_2) \tag{63}$$

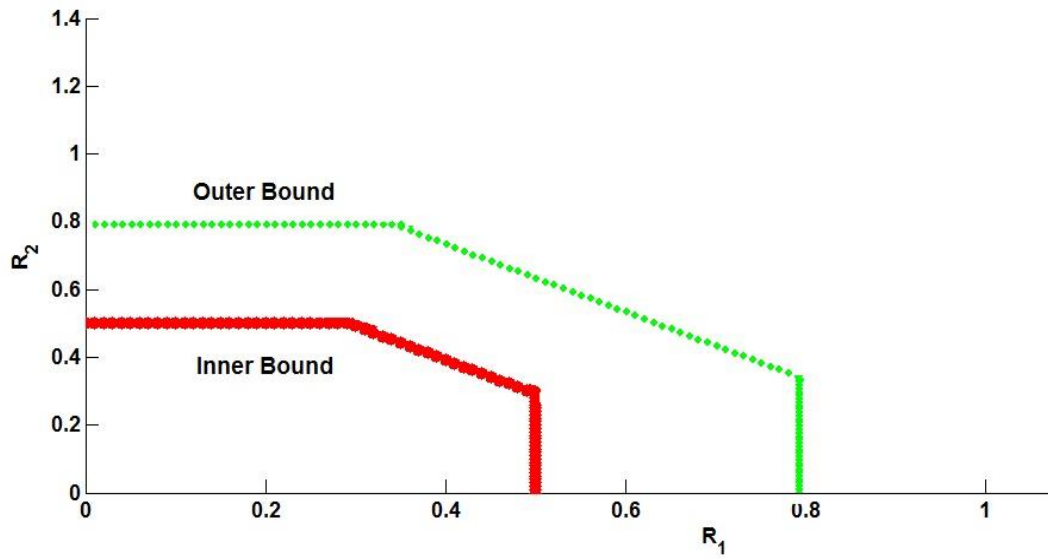$$= \frac{1}{2} \log\left(1 + P_1(1-\rho_1^2)(\frac{1}{N_1} + \frac{1}{N_2})\right) \tag{64}$$

Fig. 5. Inner and outer bound for the AWGN MARC with feedback from the receiver via two-way relay channel with

$$\frac{P_1}{N_1} = \frac{P_2}{N_1} = \frac{P_1}{N_2} = \frac{P_2}{N_2} = \frac{P_R}{N_2} = 1.$$
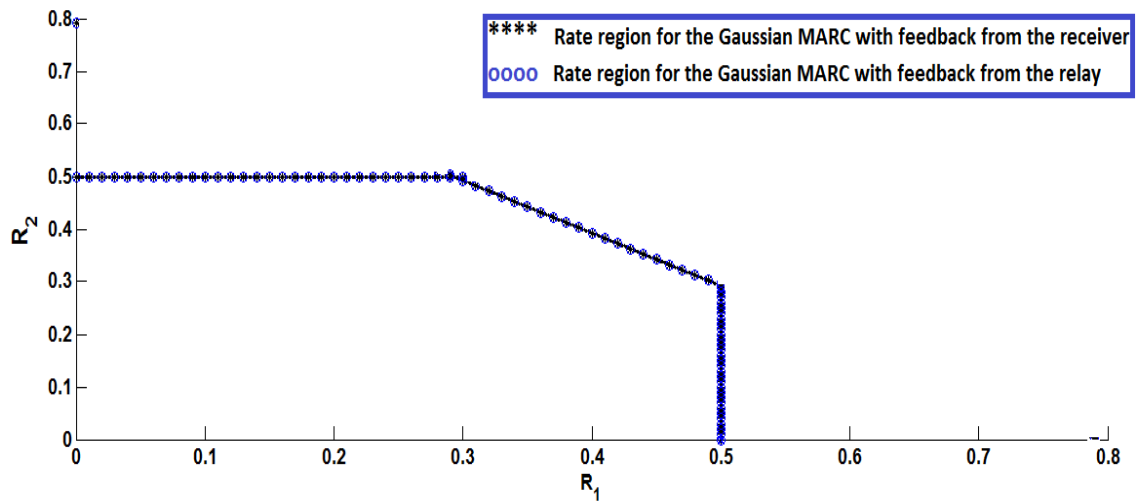


Fig. 6. Rate region for the AWGN MARC with feedback from the receiver and from the relay with

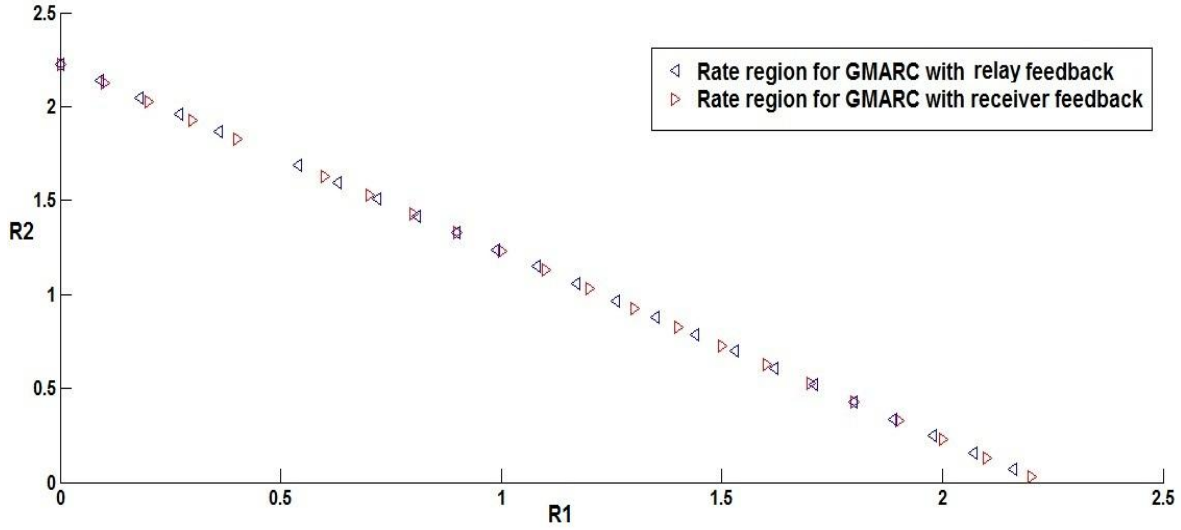$$\frac{P_1}{N_1} = \frac{P_2}{N_1} = \frac{P_1}{N_2} = \frac{P_2}{N_2} = \frac{P_R}{N_2} = 1.$$

Fig.7. Rate region for the AWGN MARC with feedback from the receiver and from the relay with

$$\frac{P_1}{N_1} = \frac{P_2}{N_1} = 30, \frac{P_1}{N_2} = \frac{P_2}{N_2} = 1 \text{ and } \frac{P_R}{N_2} = 10.$$

## VII.   NUMERICAL RESULT

With numerical results, the inner bound in (39)-(41) and outer bound in (42)-(44) for $\frac{P_1}{N_1} = \frac{P_2}{N_1} = \frac{P_1}{N_2} = \frac{P_2}{N_2} = \frac{P_R}{N_2} = 1$ are calculated and shown in Fig. 5. Our rate region for the Gaussian MARC with feedback from receiver via two-way relay channel are compared with rate in proposition 2 for $\frac{P_1}{N_1} = \frac{P_2}{N_1} = \frac{P_1}{N_2} = \frac{P_2}{N_2} = \frac{P_R}{N_2} = 1$ and $\frac{P_1}{N_1} = \frac{P_2}{N_1} = 30, \frac{P_1}{N_2} = \frac{P_2}{N_2} = 1$ and $\frac{P_R}{N_2} = 10$ (the same SNR's with [5]). These two comparing are depicted in Fig. 6 and 7. As it can be clearly seen, these two regions are accreted with each other; because, in both works relay uses decode and forward strategy and knows whatever receiver decodes.

## VIII.   CONCLUSION

We established an achievable rate region and an outer bound for the multiple access relay channels with receiver-source feedback via two-way relay channel and we extend these results to the Gaussian cases. Our achievable rate region is the same numerically in comparison with relay-source feedback.

REFERENCES

[1]  A. Sahebalam, and G.A. Hodtani, "General and new inner bound for multiple-access relay channel and two certain capacity theorems," in Communications, IET , vol.7, no.13, pp.1348-1359, September 4 2013.

[2]  G. Kramer and A. Van Wijngaarden, "On the White Gaussian Multiple Access Relay Channel," in IEEE Int. Symp. on Information Theory (ISIT 2000, Sorrento, Italy, June 25 - June 30, 2000), p. 92.

[3]  N. Gaarder and J.wolf, "The capacity region of a multiple-access discrete memoreyless channel can increase with feedback," in IEEE Trans. Inform. Theory, vo. 21, no. 1, pp. 100-102, Jan. 1975.

[4]  S. I. Bross and M. A. Wigger, "On the Relay Channel With Receiver-Transmitter Feedback ," in IEEE Trans. Inform. Theory, vo. 55, no. 1, pp. 275-291, Jan. 2009.

[5]  J. Hou, R. koetter and G. Kramer, "Rate Regions for Multiple Access Relay Channels with Relay-Source Feedback," in IEEE Information Theory Workshop (ITW 2009, Oct. 2009), pp. 288-292.

[6]  C. E. Shannon, "Two-way communication channels," in Proc. 4th Berkeley Symp. Math. Stat. and Prob., vol. 1, pp. 611-644, 1961.

[7]  B. Rankov and A. Wittneben, "Spectral Efficient Signaling for Halfduplex Relay Channels," in Proc. of Asilomar Conference on Signals, Systems and Computers, pp. 1066-1071, Oct. 2005.

[8]  B. Rankov and A. Wittneben, "Achievable Rate Regions for the Two-Way Relay Channel," in Proc. of IEEE ISIT, pp. 1668-1672, July 2006.

[9]  C. Hausl and J. Hagenauer, "Iterative Network and Channel Decoding for the Two-Way Relay Channel," in Proc. of IEEE ICC, pp. 1568-1573, June 2006.

[10] S. Zhang, S. C. Liew, and P. P. Lam, "Hot Topic: Physical-Layer Network Coding," in Proc. of ACM Mobicom, pp. 358-365, 2006.

[11] T. Cui, F. Gao, T. Ho, and A. Nallanathan, "Distributed Space-Time Coding for Two-Way Wireless Relay Networks," in Proc. of IEEE ICC, May 2008.

[12] D. Gunduz, E. Tuncel and J. Nayak, "Achievable Rate Regions for the Two-Way Relay Channel in Different Strategies," in Proc. of 46th Annual Allerton Conf. on Comm., Control, and Computing, Monticello, IL, Sept. 2008.

[13] T. Cover and J. Thomas, Elements of Information Theory, Wiley, 2006.