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Extraction of Electromagnetic Scattering from Random Rough Surfaces in Complex Environments using Numerical Methods

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Abstract- **In this study, we present a comprehensive overview of the exact and approximate models and methods that are available to measure the dispersion of random rough surfaces. We investigate the structural implementation of such surfaces in the finite-difference time-domain (FDTD) simulation environment and introduce a method for error reduction in simulated environments with such surfaces to improve the accuracy of near-path scattering of random rough surfaces. For this aim, a new adsorbent consisting of two types of adsorbents with distinct properties is proposed. During different tests in environments with the random rough surfaces and more complex environments with abnormal dispersers, the superiority, and higher performance of the proposed adsorbents are verified and then, properties of the adsorbents are investigated. Next, two dimensional random rough surfaces are analyzed to investigate electromagnetic scattering. To determine the electromagnetic scattering field, surface height's and slope's joint probability density function is calculated and utilized after generating a two-dimensional rough surface. The ray-tracing base model is exploited, and then the Monte-Carlo technique is hired to convert an infinite integration form into the form of finite integration.**

Index Terms- Absorbent layers, Random rough surface scattering, Finite-difference time-domain (FDTD) numerical method, Numerical methods in electromagnetism

I. INTRODUCTION

Recently, the scattering of electromagnetic by two-dimensional random rough surfaces has gathered a lot of attention [1-11]. Due to the impact of this phenomenon in various disciplines of sciences such as geophysics, remote sensing of the earth, and communication, nowadays, analysts are excited to solve such

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problems. Conventional approaches that are utilized for two-dimensional random rough surface scattering are Kirchhoff approximation and small perturbation [1], [2]. The drawback of utilizing the mentioned methods is the limitation in their domain of validity. Due to the advancement of fast numerical methods and the advent of modern computers, Monte Carlo simulations of direct solutions for electromagnetic scattering problems are popular [9-10], [15] & [17]. Monte Carlo's procedure is also restricted by its requirements for a more powerful simulation method and the usage of a long surface. Regarding the expansion of knowledge and information technology with developing computer resources simultaneously, simulation of natural phenomena nowadays has become one of the most widely used and challenging topics for researchers and engineers. Due to the superfluous usage of high-frequency devices, precise simulation of random rough surfaces and their effects on the electromagnetic waves are also among the issues that are interesting to researchers [11-21]. Improving computer resources increasingly allows us to simulate and study surface scattering on a relatively large scale and extract their features.

In this research, the electromagnetic scattering field is investigated through the joint PDF of height and slope. The surface height and slope are supposed to be uncorrelated. In the first step, a 2D random rough surface is generated by using MATLAB. The ray-tracing method, using [3], is hired to explore the electromagnetic field. Then, by referring to the two-dimensional configuration in polar coordinates, the 2D surface slope is calculated. We determined the joint PDF of the slope and height after acquiring the surface slope, and finally, the PDF of the height is calculated.

II. FINITE DIFFERENCE TIME DOMAIN METHOD (FDTD)

In the last decade, the FDTD method has gained popularity as a management tool in solving Maxwell equations [9]. This method is based on simple formulations without the demand for Green's approximations and functions. Besides solving problems in the time domain, one can easily investigate the frequency response over a wide range of frequencies using the Fourier transform. The prevalence of this numerical method is due to the ability to simulate environments, including dielectric, nonlinear materials, frequency-dependent materials, and non-identical materials. In recent years, with the development of computer resources and the high potential of this method, it has been possible to implement this method by parallel processing.

III. GENERATION, SIMULATION, AND EXTRACTION OF RANDOM-SURFACE SCATTERING USING FDTD METHOD

A. 2D Surface Model

Except for height function to the two-dimensional surfaces, producing rough two-dimensional surfaces is the same as the one-dimensional model, $z= (x.y)$ is a Gaussian process. This Gaussian process is also fully characterized by the correlation function < $(x_1.y_1)$><f $(x_2.y_2)$> = h_1C $(x_1.y_1; x_2.y_2)$, where h_1 is the effective surface area.

B. Similar Mode

The spectral density of the power is $(kx.ky)$. Then, consider the function $(x.y)$ outside of the alternating $Lx\times$: $(x.y) = f\ (x+Lx.y+Ly)$ by assuming $Lx\times Ly$ dimension for the surface. As a result, we can use the Fourier series to represent $f\ (x.y)$:

$$f(x.y) = \frac{1}{L_xL_y}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} F_{mn}(k_{xm}.k_{yn})e^{-(k_{xm}x+k_{yn}y)} \tag{1}$$

Where 2 = $2\pi mLx$ and 2 = $2\pi nLy$. Similar to the one-dimensional case, the Fourier series coefficients F_{mn} $(k_{xm}.k_{ym})$ are obtained using the spectral density of the power, and thus the surface height function is determined. Accordingly, the relationships related to the calculation of the Fourier series coefficients are given below:

$$F_{0.0} = 2\pi\sqrt{L_xL_yW(0.0)r_a} \tag{2}$$

$$F_{0.\frac{N}{2}} = 2\pi\sqrt{LxLyW(0.\frac{\pi N}{Ly})r\beta} \tag{3}$$

$$F_{\frac{M}{2}.0} = 2\pi\sqrt{L_xL_yW(\frac{\pi M}{L_x}.0)r_\gamma} \tag{4}$$

$$F_{\frac{M}{2}.\frac{N}{2}} = 2\pi\sqrt{L_xL_yW(\frac{\pi M}{L_x}.\frac{\pi N}{L_y})}r_\rho \tag{5}$$

$$F_{mn} = F^{\cdot}_{-m-n}, F_{M/2n} = F^{\cdot}_{-m.N/2} \tag{6}$$

After calculating the values of F_{mn}, using the equations, the Rough Rise Height Profile will be calculated by the inverse fast Fourier transform (IFFT). In the two-dimensional case, the Gaussian correlations function with its corresponding spectral density.

$$c(x.y) = h^2e^{-(\frac{x^2}{cl_x^2}+\frac{y^2}{cl_y^2})} \tag{7}$$

$$W(k_x.k_y) = \frac{h^2cl_xcl_y}{4\pi}e^{-(\frac{k_x^2cl_x^2}{4}+\frac{k_y^2cl_y^2}{4})} \tag{8}$$

And the exponential correlation function along with its corresponding spectral density is:

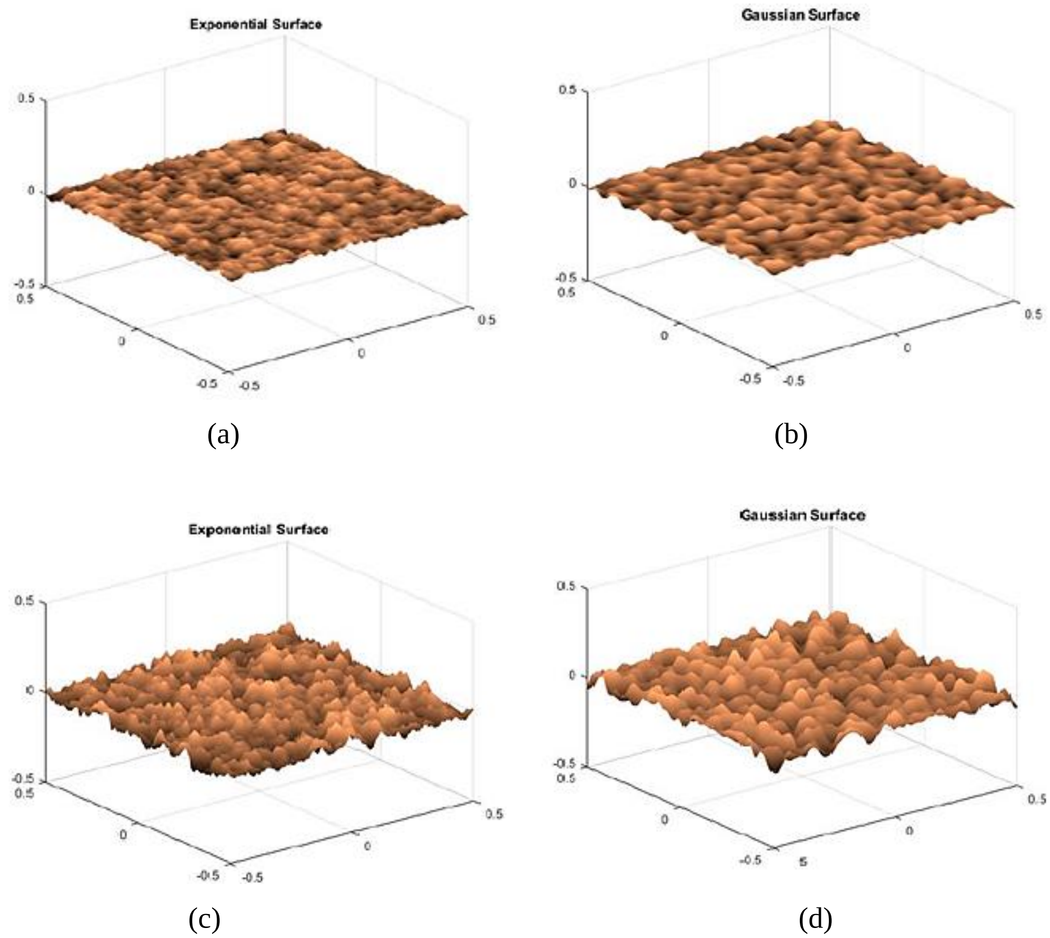


Fig. 1. (a) Exponential surface (h=0.01), (b) Gaussian surface (h=0.01), (c) Exponential surface (h=0.03) and (d) Gaussian surface (h=0.03).

$$c(x.y) = h^2 e^{-\sqrt{\left(\frac{x^2}{cl_x^2}+\frac{y^2}{cl_y^2}\right)}} \tag{9}$$

$$W(k_x.k_y) = \frac{h^2 cl_x cl_y}{2\pi}\left[1 + (k_x^2 + k_y^2) cl_x cl_y\right]^{-\frac{3}{2}} \tag{10}$$

IV. INSERTING A RANDOM UNEVEN SURFACE INTO THE FDTD NETWORK ENVIRONMENT

All quantities, including field components and environmental parameters in the FDTD method, are entered into discrete matrices and update equations of the FDTD or YEE’s method [4]. These matrices must be specified to be an accurate model of the under-consideration environment's characteristics. Besides, the dispersion property can be added to the simulation environment by

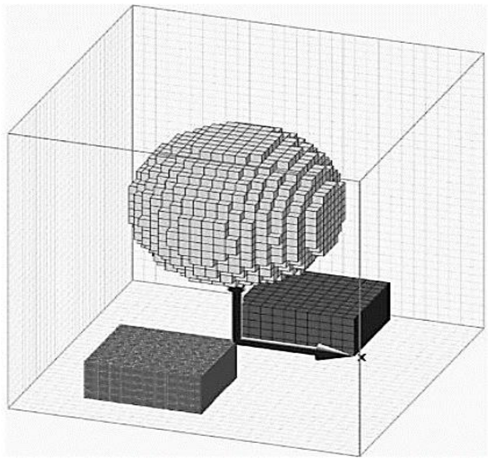


Fig. 2. Approximation of some simple objects in the FDTD network environment. As can be seen, the curvature of the FDTD method is subjected to very small spatial steps, which are important for fine grading (due to the existing computer constraints it is certainly not feasible).

considering the frequency dependence of the above parameters. All the parameters and properties of the investigated materials will be networked due to the network structure of the FDTD method. Fig. 2 shows an overview of this network structure and the shape of the simulated objects in this environment.

V. EXTRACTION OF SCATTERING FIELDS IN REFLECTION AND TRANSFER BY KA METHOD IN 3D

After a thorough two-dimensional study $(x.z)$, we turned to the three-dimensional problem in which the surface is defined as $(x.y)$ with elevation changes in two directions x- and z- axes. The concepts and calculations are explained in detail for the two-dimensional case, then, for the three-dimensional problem, only the main steps are discussed.

As can be seen in Fig. 3, (R) = $E0\exp$ $(iK_i.R)$ $\hat{e_i}$ represents the electric field radiating in the direction of $\hat{K}i$= $(\hat{K}_i x.\hat{K}_i y.\hat{K}_i z)$ = $(K_i x.K_i y.K_i z)$/$|K1|$ on the surface. The angle of height radiation is θ_i concerning the z-axis, and the angle of radiation is $2\varphi_i$ to the x-axis. Likewise, Es represents the scattered field in the direction of $\hat{K}s$= $(\hat{K}sx.\hat{K}sy.\hat{K}sz)$ = $(Ksx.Ksy.Ksz)$/$|Ka|$ with scattering angle θs in height and φs in the side. In reflection mode, S and α are equal to r and 1, respectively. However, for transmitting mode, S and α are equal to t and 2, respectively. The perpendicular unit vector to the surface at point A is calculated in the three-dimensional state using Equation (11).

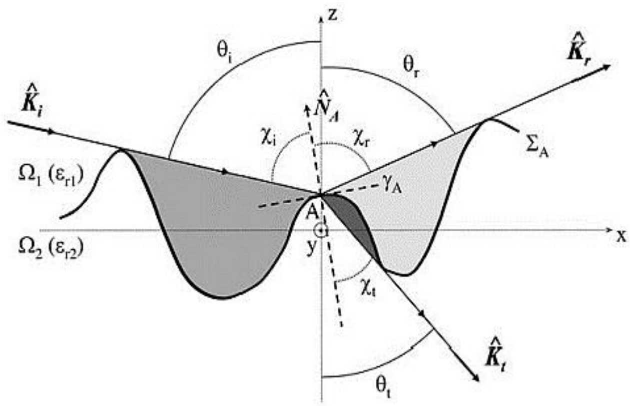


Fig. 3. Displays the three-dimensional problem on the page - (***x.z***) with a constant y.

$$\hat{N}_A = \frac{-\gamma_{A.x}\hat{x} - \gamma_{A.y}\hat{y} + \hat{z}}{\sqrt{1+\gamma_{A.x}{}^2+\gamma_{A.y}{}^2}} \tag{11}$$

Where $\gamma A.x{=}\partial\xi A/\partial xA$ and $\gamma A.y{=}\partial\xi A/\partial yA$ are the local slopes in x- and y-axes at point A, respectively. As in the previous section, this model is based on the integral equation on the surface. In other words, this model is based on equations called Kirchhoff and Helmholtz equations which calculate the scattering fields in the reflection-transition state. Note that, $(R.RA)$ is a three-dimensional green function calculated using Equation (12).

$$\bar{G}_a(R.R_A) = (\bar{I} + \frac{\nabla\nabla}{k_a^2})G_a(R.R_A).\, G_a(R.R_A) = \frac{e^{ik_a\|R-R_A\|}}{4\pi\|R-R_A\|} \tag{12}$$

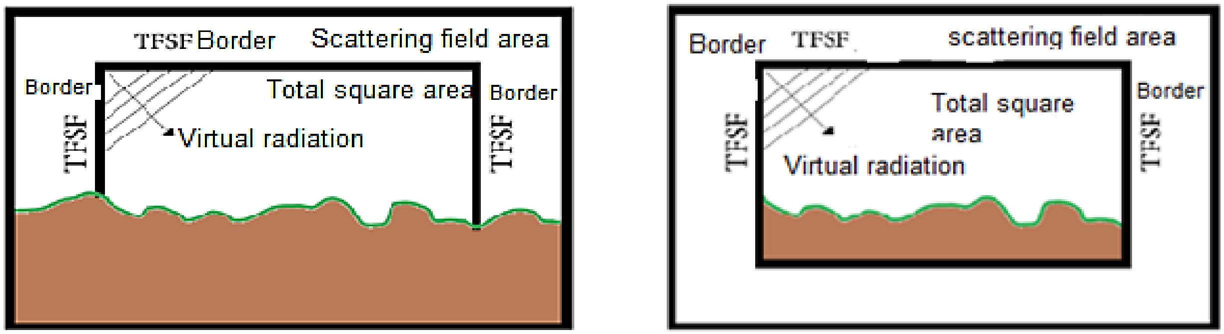
For distance approximation we know:

$$\bar{G}_a(R.R_A) \simeq (\bar{I} + \hat{K}_s\hat{K}_s)\frac{e^{ik_a(R-\hat{k}_s.R_A)}}{4\pi R} \tag{13}$$

The following vector relations are also useful for the telecommunication field, which allows us to write equations for scattered fields at a distance:

$$\nabla \times \bar{G}_a(R.R_A) \simeq -ik_a k_s \times G_a(R.R_A) \tag{14}$$

Following equations are a reference for applying the KA method:



(a) (b)

Fig. 4. Two methods adopted to solve the error problem caused by surface edge effects.

$$E_r^{\infty}(R) = +\frac{ik_1 e^{ik_1 R}}{4\pi R}(\bar{I} - \hat{k}_r\hat{k}_r).\iint \sum A\, d \sum A\{\eta_1[N_A \times H(R_A)] + \hat{k}_r \times [N_A \times E(R_A)]\}e^{-ik_r.R_A} \quad (15)$$

$$E_t^{\infty}(R) = +\frac{ik_2 e^{ik_2 R}}{4\pi R}(\bar{I} - \hat{k}_r\hat{k}_r).\iint \sum A\, d \sum A\{\eta_2[N_A \times H(R_A)] + \hat{k}_t \times [N_A \times E(R_A)]\}e^{-ik_r.R_A} \quad (16)$$

VI. ERROR CAUSES IN MEASURING SCATTERING FIELDS NEAR RANDOM ROUGH SURFACES

Investigation and extraction of random rough surfaces and shooting of objects in the presence of such rough surfaces require high accuracy of simulation and minimization of available errors. Particularly, there is a great reduction in the return waves due to the high conductivity of the environment (the ambient is predominant). The presence of random rough surfaces at different stages of the simulation environment creates conditions that reduce the accuracy of near field measurements. Therefore, the accuracy enhancement of the applied numerical method to investigate random rough surfaces will improve the accuracy of the results.

VII. EFFECT OF ROUGH SURFACE EDGES

The limited FDTD simulation space produces errors for the unlimited length of rough surfaces study. As a result, the implemented surfaces are usually cut to the side or stretched across the absorber boundary. These two cases are depicted in Fig. 4.A and 4.B, respectively.

In Fig. 4(a), the TFSF environment rough surface, the absorbent environment, and the NTFF environment (if any) intersect, which makes the problem slightly different from common simulation problems. Consequently, the boundaries of the TFSF, adsorbent, and the NTFF in which they are located around the scattered. In Fig. 4. (b), where the surface is cut off on both sides, the diffraction created by the rough surface edges causes an error in the simulation environment. It is important to note that although the narrow beam field plays an effective role in minimizing this error, the unique shape of the random rough surface and its random structure in this case also increases the error of the adsorbents. Both of these methods induce errors, and thus, they challenge the measurement of near-field fields around random rough surfaces. However, method 4.A is preferred due to the absence of edge effects.

A. 2D Rough Surface Generation

In this segment, we produced a two-dimensional random rough surface. Several approaches have been established to engender random rough surfaces. These include lines-oriented and convolution methods for producing heterogeneous random rough surfaces and are conducted by Fourier transform [4], [5], [6]. In this essay, parameters of the rough surface are derived analytically by means of the convolution technique for its generation.

B. 2D Rough Surface Characteristics

Generally, a surface is considered by its autocovariance, spectrum density, autocorrelation function, and its correlation length. The Gaussian height probability density function, Equation (17), is specified by [7], [8].

$$P(Z = f(x,y)) = \frac{1}{\rho\sqrt{2\pi}} e^{\frac{(z=f(x,y))^Z}{z\rho^2}} \tag{17}$$

Where $(x,\ y)$ and ρ^2 signify the height function of the 2D random rough surface and its mean square, respectively. Spectrum density function ($\boldsymbol{K}$) for the 2D surface is defined as follows:

$$W(k) = \left(\frac{el^2\rho^2}{4\pi}\right) e^{\frac{k^2cl^2}{4}} \tag{18}$$

where cl is the correlation length and the spatial angular frequency vector ($\boldsymbol{K}$) is defined by:

$$k = (k_x, k_y), k = \sqrt{k_x^2 + k_y^2} \tag{19}$$

Using the spectrum density function, we should ascertain the autocorrelation function (ACF).

$$ACF = \int_{-\infty}^{+\infty} W(k) e^{ik(x+y)} dk = h^2 e^{\frac{-x^2}{el_x^2} - \frac{y^2}{el_y^2}} \tag{20}$$

where h, $clx\ and\ cly$ are the RMS height, correlation lengths in x- and y-axes, respectively. The slope will be Gaussian since both height and ACF are Gaussian distributions. The slope in $x\text{-}\ and\ y$-axes give:

Table 1. 2D rough surface parameter values.

Parameters	**Number of surface points**	**rL Length of surface**	**h RMS height**	**Clx and Cly Correlation length (in x- and y-axes)**
Values	400	1 λ	0.09 λ	0.06 λ

$$S_x = \frac{\partial f}{\partial x} and S_y = \frac{\partial f}{\partial y} \tag{21}$$

Finally, we obtain the power spectrum density function of the surface where the RMS slope for the Gaussian rough surface is presented by w, which is equal to $\rho\sqrt{2}\ cl$.

$$Ps(s_x, s_y) = \frac{1}{2\pi w_x w_y} e^{\frac{-s_x{}^2}{2w_x^2} - \frac{s_y{}^2}{2w_y^2}} \tag{22}$$

VIII. ALGORITHM AND PLOT FOR 2D ROUGH SURFACE GENERATED

In the convolution technique, the 2D Discrete Fourier transform (DFT) is specified by $F = DFT\ (f)$ in which the inverse DFT is $f = DFT^{-1}\ (f)$.

$$F_{\gamma_x \gamma_y} = \sum_{n_y=0}^{N_y-1} \sum_{n_x=0}^{N_x-1} f_{n_x n_y} e^{-j2\pi(\frac{n_x \gamma_x}{N_x} + \frac{n_y \gamma_y}{N_y})} \tag{23}$$

where γ_x and γ_y are equal to 0, 1, …, N_x-1 and 0, 1, …, N_y-1, respectively.

For 2D case, Nx *and* Ny represent the numbers of arrays in x- *and* y-*direction*, respectively. Therefore, we have the surface height as Equation (24). MATLAB simulating software is utilized to perform the generation of a 2D surface. A two-dimensional random rough surface is depicted in Fig. 5, which is derived analytically by using the convolution method. Furthermore, the Probability Distribution Function (PDF) of 2D rough surface height is also illustrated in Fig. 6 by using of MATLAB simulator.

$$f_{n_x n_y} = \sum_{\gamma_y=0}^{N_y-1} \sum_{\gamma_x=0}^{N_x-1} F_{\gamma_x \gamma_y} e^{j2\pi(\frac{n_x \gamma_x}{N_x} + \frac{n_y \gamma_y}{N_y})} \tag{24}$$

where n_x and n_y are equal to 0, 1, …, N_x-1 and 0, 1, …, N_y-1, respectively.

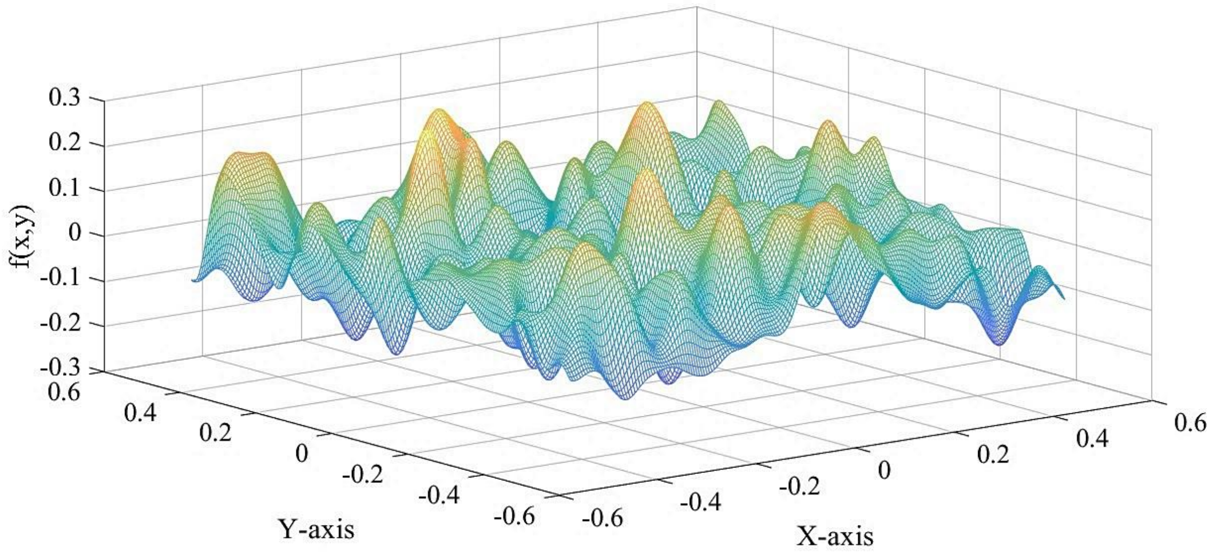


Fig. 5. Two-dimensional rough surface generated.

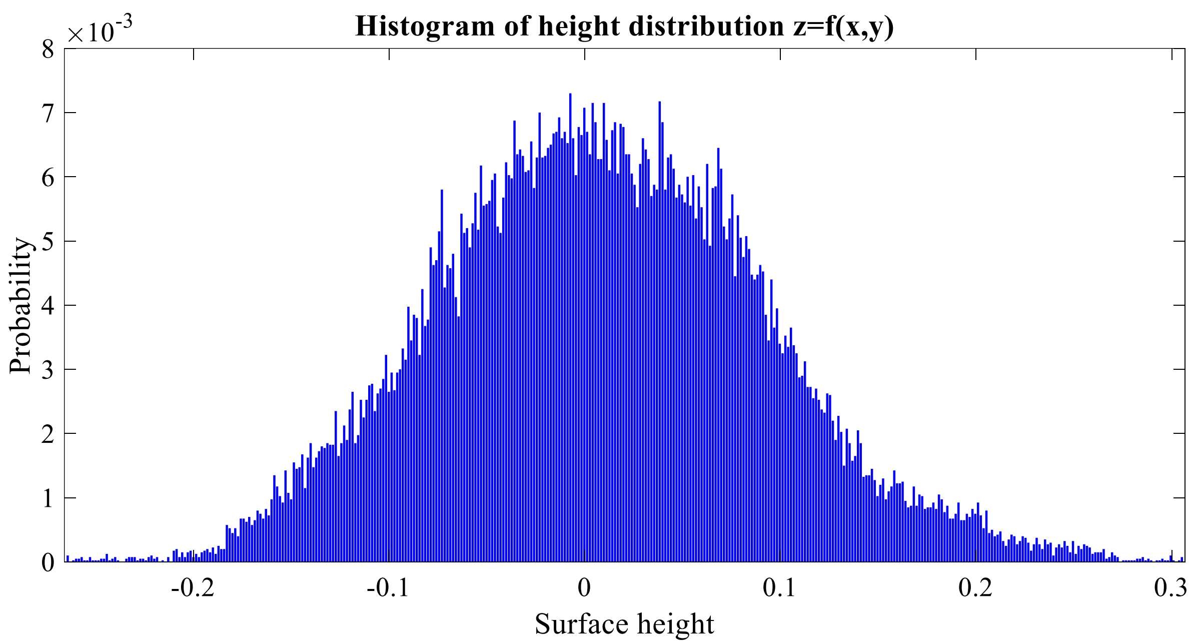


Fig. 6. PDF of 2D rough surface height.

IX. CONCLUSION

In this research, using the joint probability distribution function of the height and slope of a 2D rough surface, a new method to compute the electromagnetic scattering field has been proposed. The proposed method in [9] has been compared with this method. The contrast can be notified between the two methods. But, the proposed method in this essay is more precise due to its satisfactory agreement with others' researches results in the literature. For our case study of $Psz\ (S, Z)$, we can discern from the Fig., the scattering field increases swiftly with the scattering angle, and it got an inflection point at around 80-degree. From the 80-degree point, it begins to increase by 20 dB slope until it reaches a critical point at around 100-degree scattering angle, where it starts to decrease. In contradiction to our approach, for the recommended technique during the case study using $P2(\theta s)$, the scattering field is almost persistent (0 dB), and at 80-degree of scattering angle, it gains approximately 10 dB before it decreases about 5 dB. Finally, it is almost constant (5 dB). There is an appropriate agreement between the results of our technique and those which are offered in the literature. These include Kirchhoff Approximation, Monte Carlo Simulation [10], and the approach used in [11]. The results have demonstrated the accuracy and effectiveness of our proposed method.

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